A Simple and Efficient Error-Diffusion Algorithm

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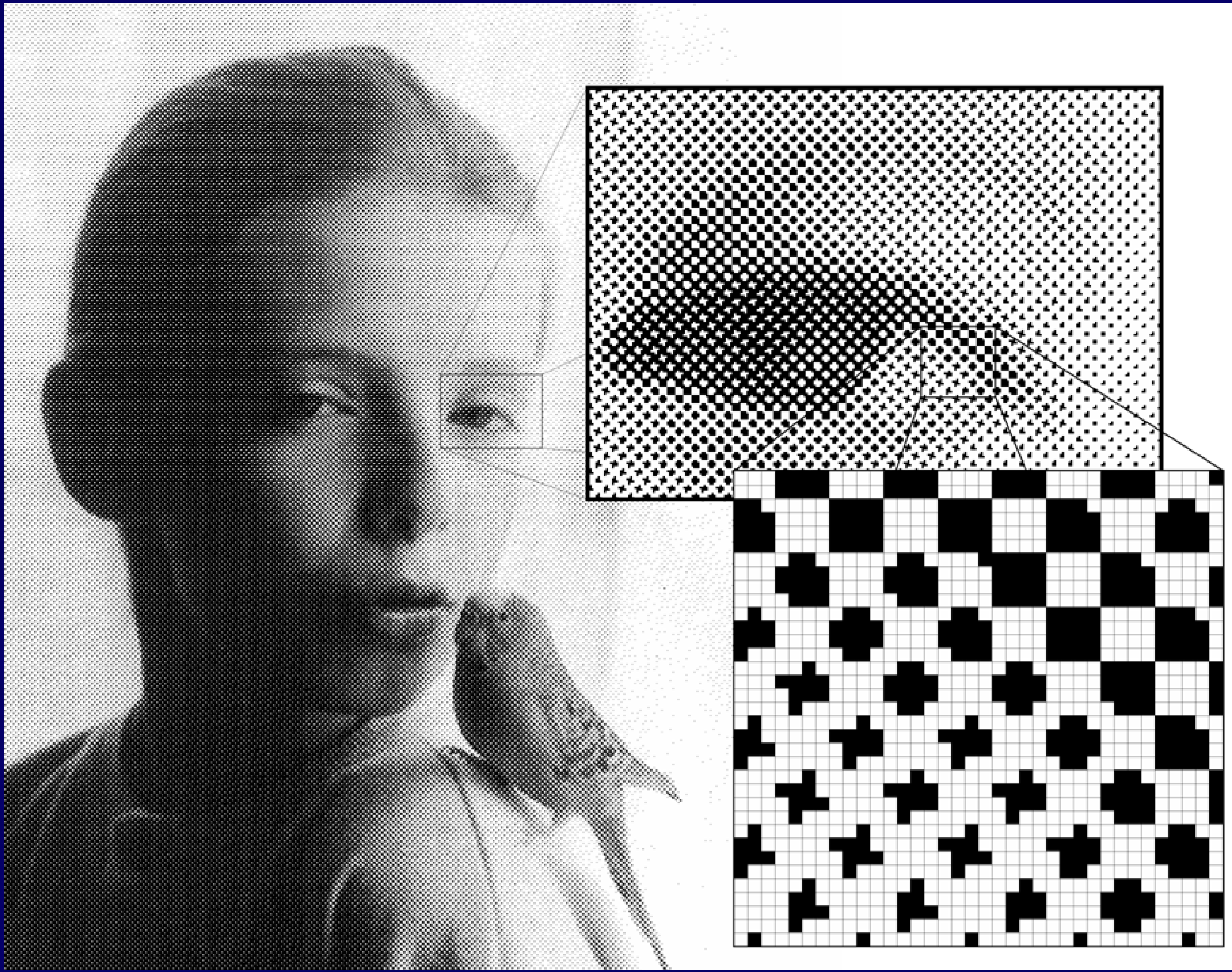
Motivations

- Error-Diffusion is Important Visualization
- No Satisfactory Solution Exists Today
- A Simple and Efficient Solution is Possible

Outline

- Introduction
- Problem Statement
- Our Variable-Coefficient Error-Diffusion Algorithm
- Results
- Conclusions and Future Work

What is Halftoning?



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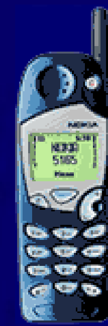
Where Do We Need Halftoning?

Displays

PDAs, Mobile Phones



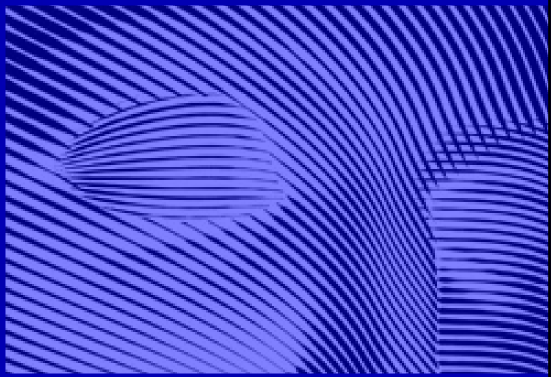
Games

Target Devices for Our Algorithm: Low Resolution, Where Individual Pixels Are Visib

Printers

Network-Based Imaging

Artistic Rendering



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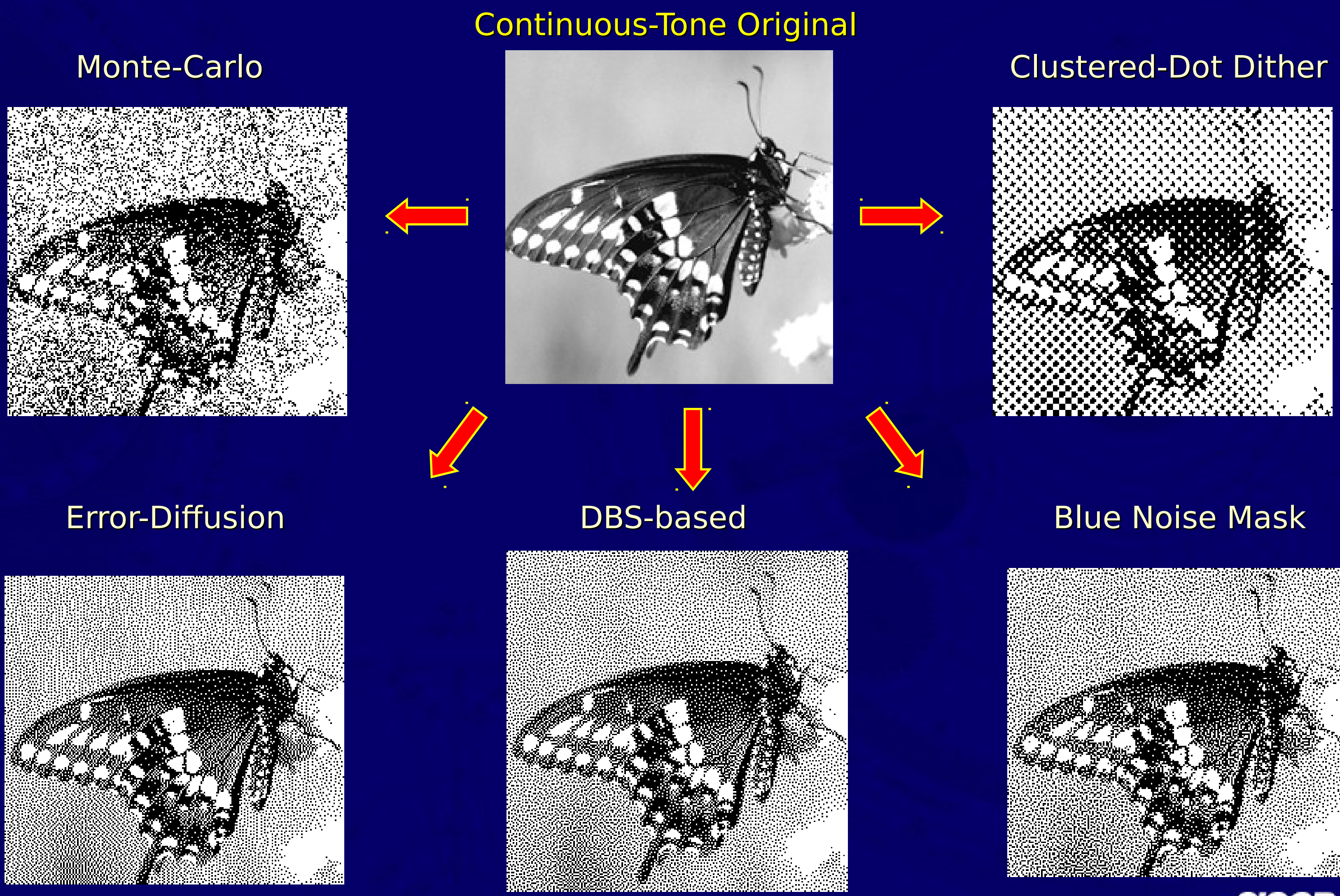
Halftoning: Basic Classification



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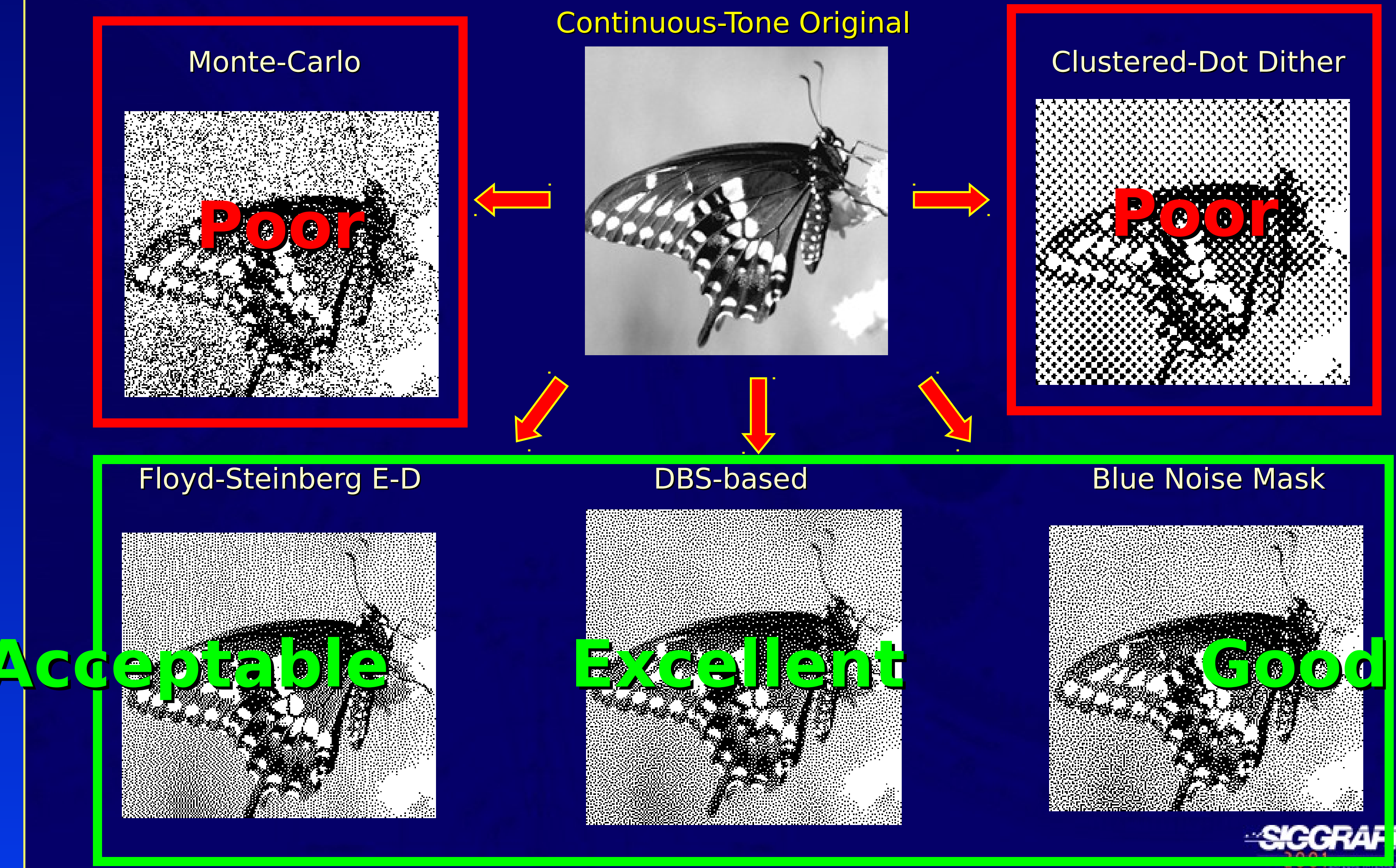
Halftoning Algorithms: General Requirements

- Visual Quality
- Execution Speed
- Conceptual Simplicity
- Legal Availability

Criterion: Visual Quality for Low-Resolution Displays



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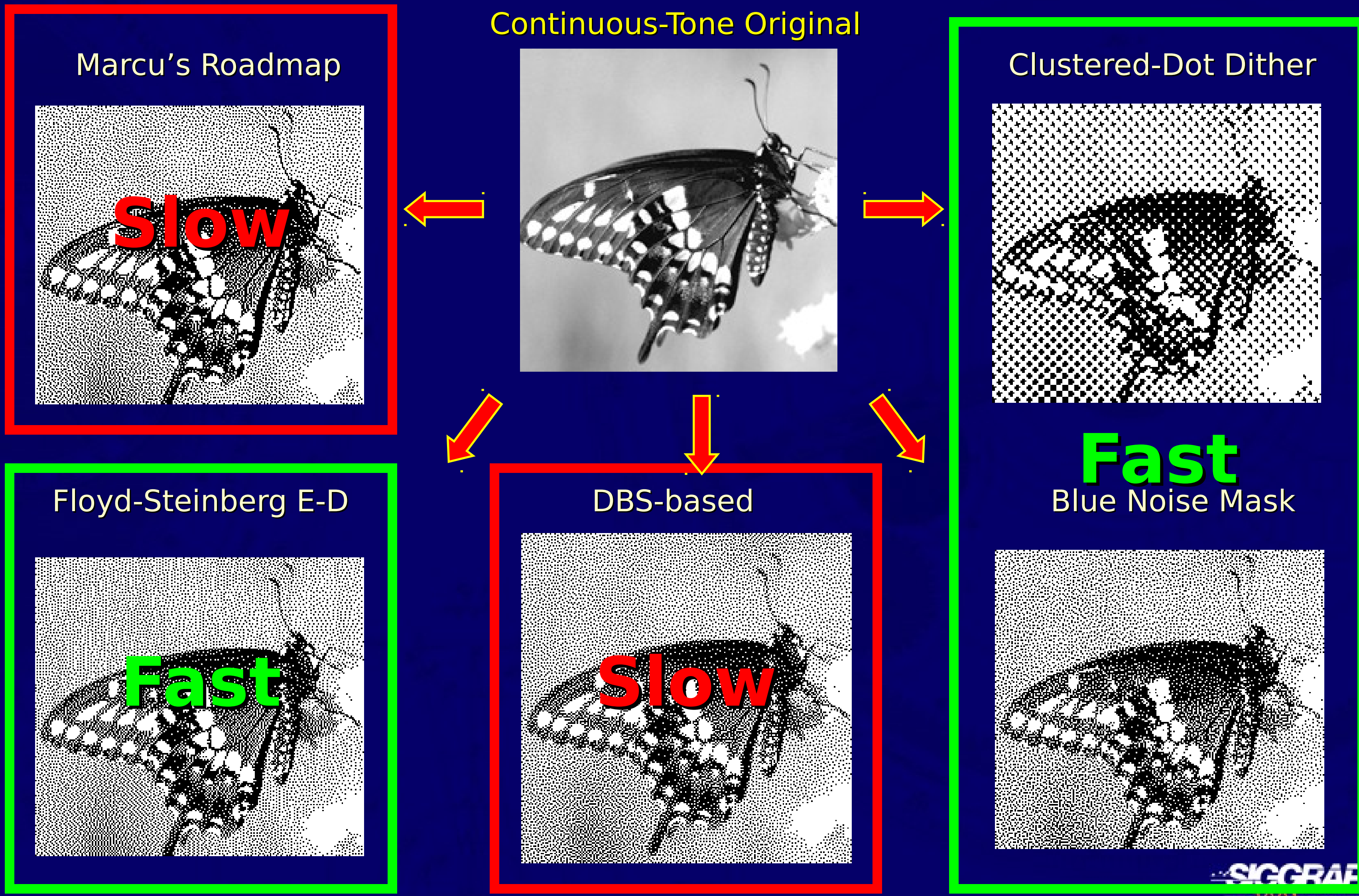
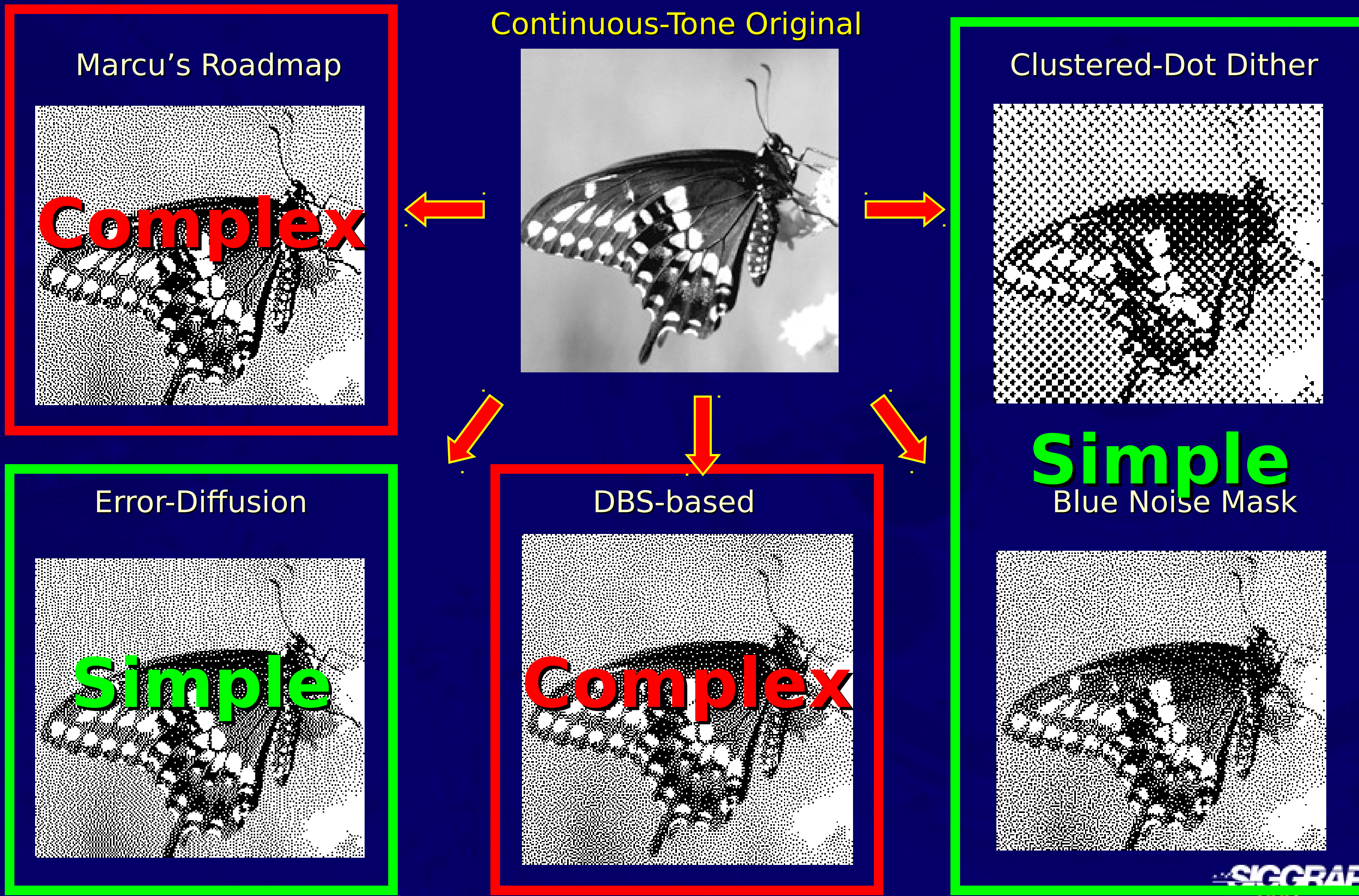
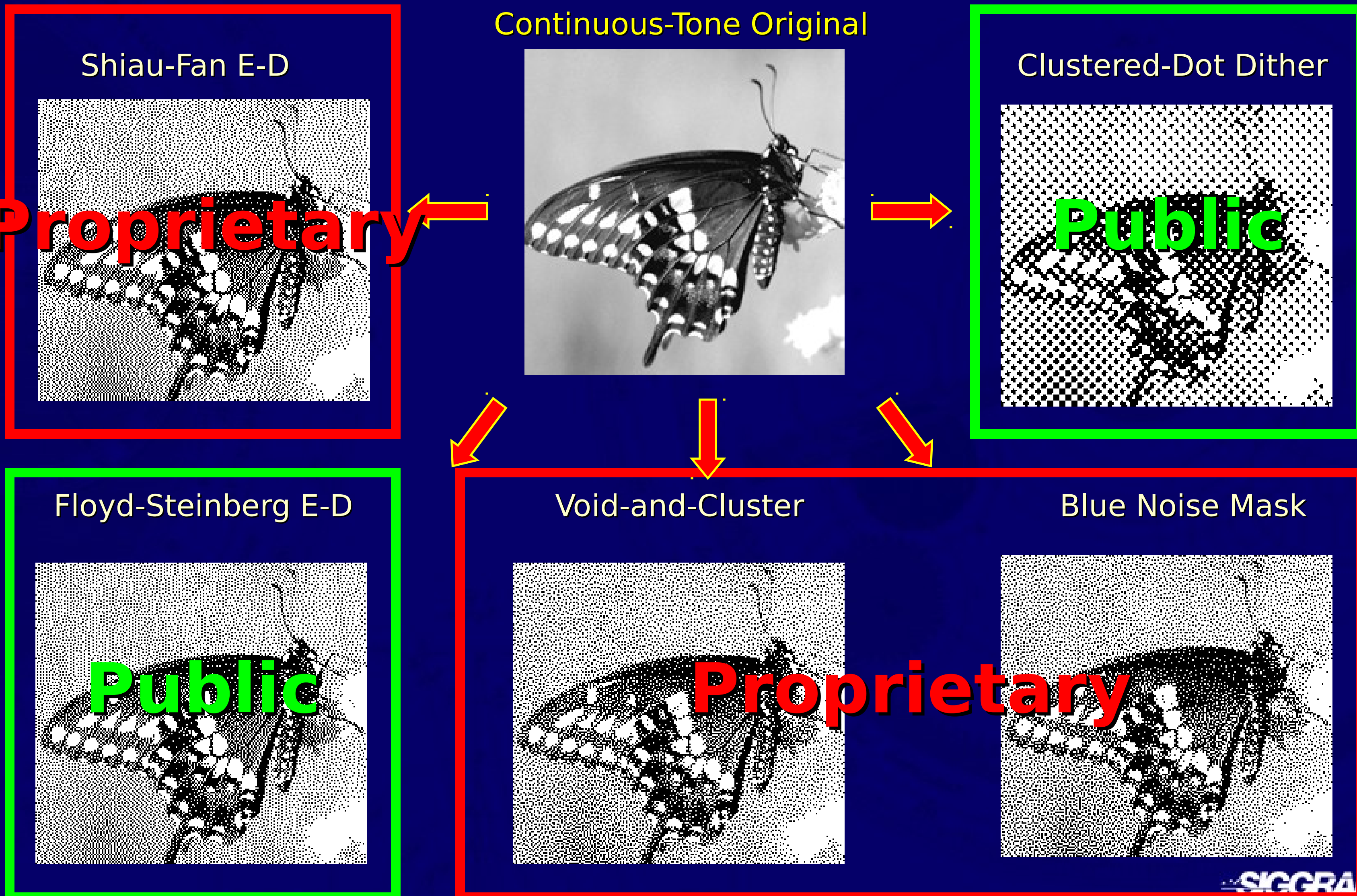
Criterion: Execution Speed



Criterion: Conceptual Simplicity



Criterion: Public Availability



State-of-the-Art Status

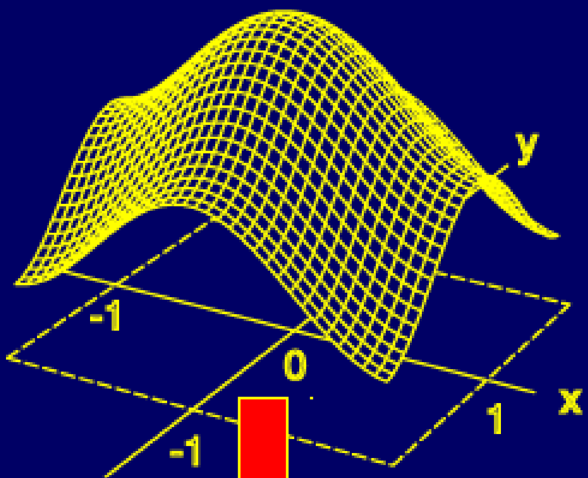
There is NO Halftoning Algorithm that Wold Fully Satisfy All Criteria

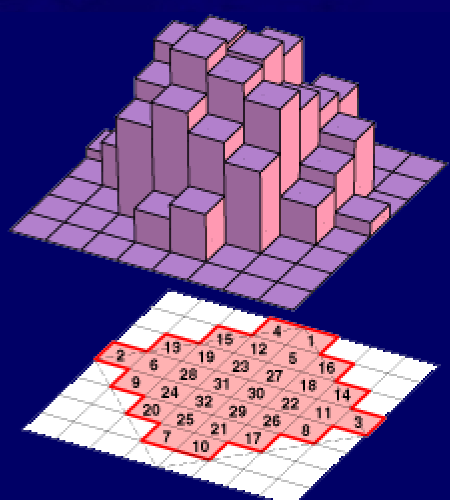
- **Visual Quality**
- **Execution Speed**
- **Conceptual Simplicity**
- **Legal Availability**

➔ Main Motivation for Our Work

Clustered-Dot Dithering

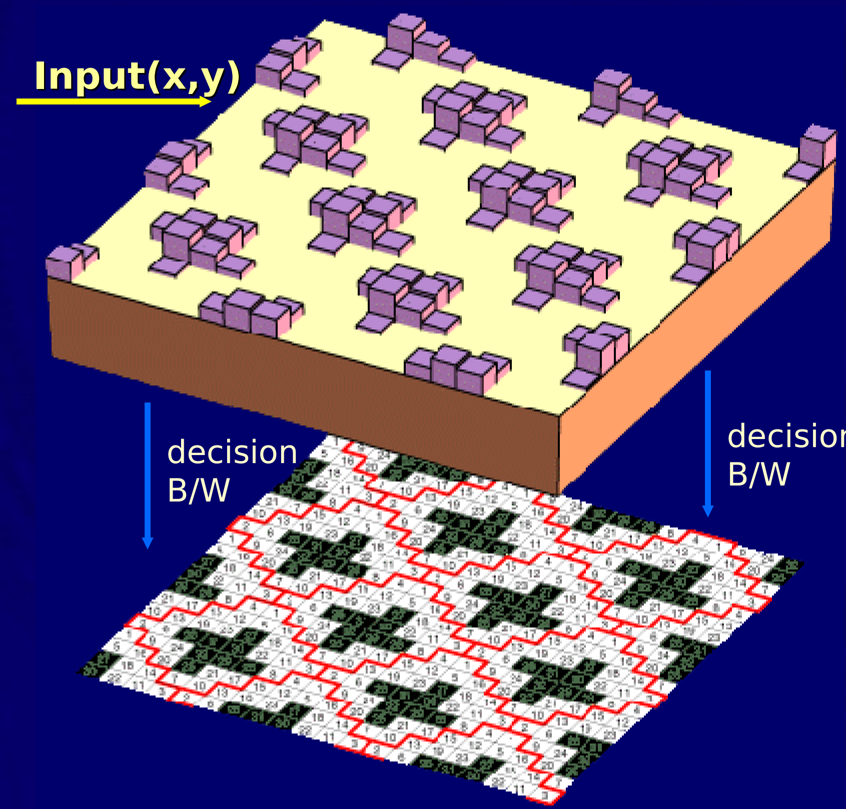
Spot Function (here: Egg-crate function)



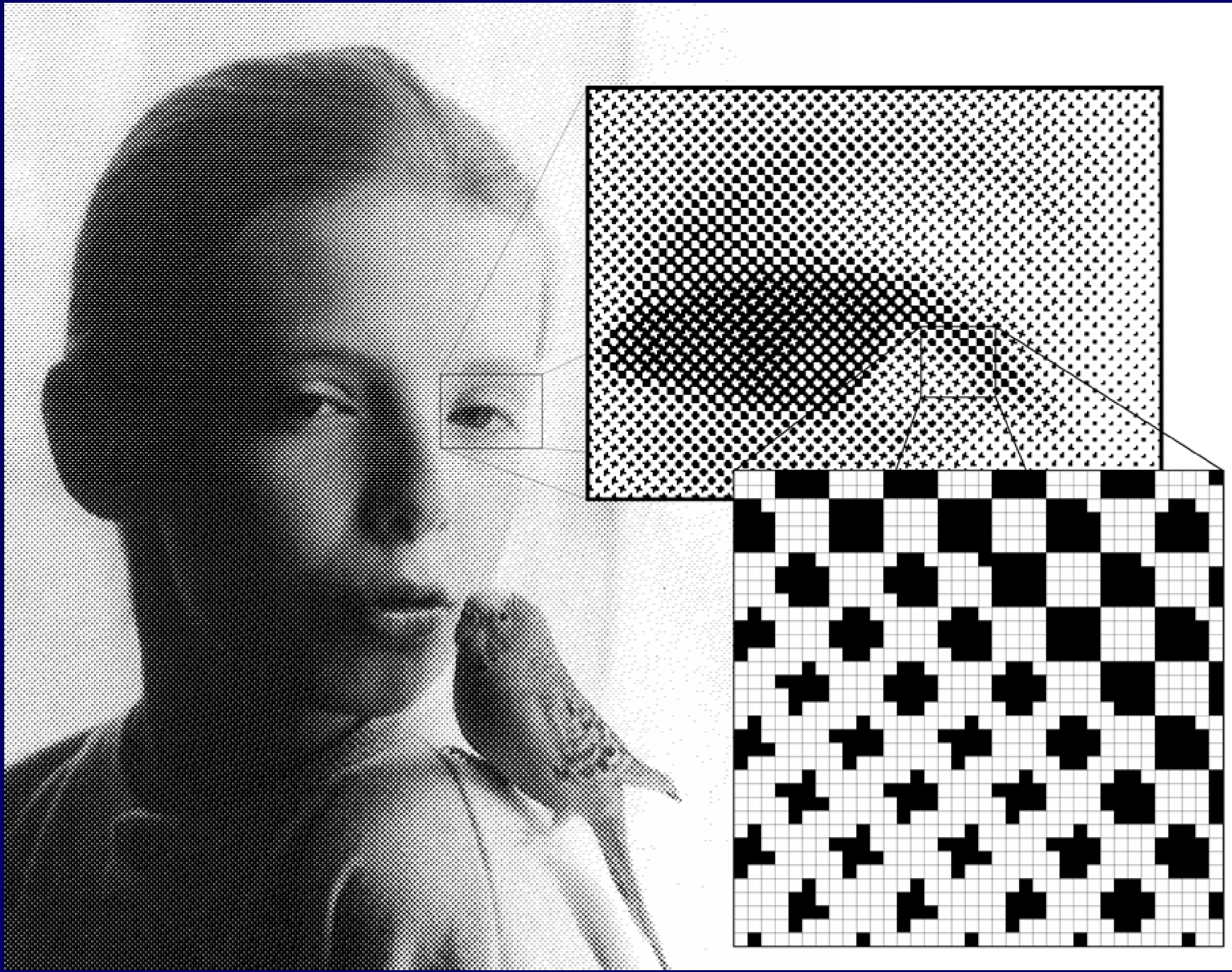


Threshold(x,y)

Compare **Threshold(x,y)** and **Input(x,y)**

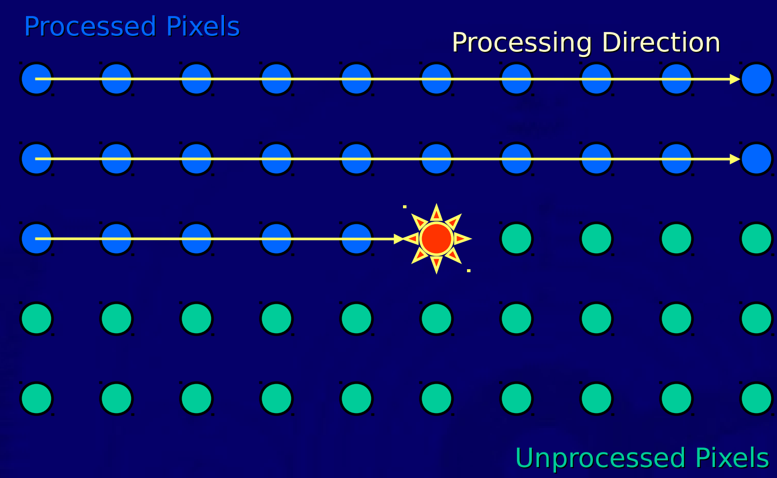


Clustered-Dot Dithering



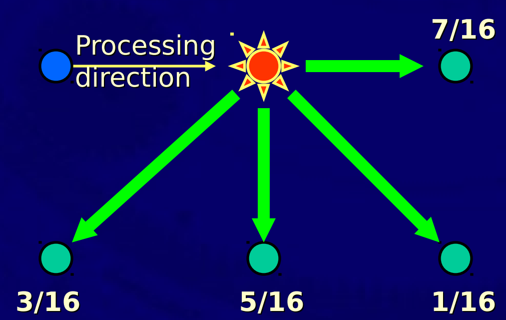
Floyd-Steinberg Error-Diffusion Algorithm (1975)

- **Process Pixels in Order of Scanlines**



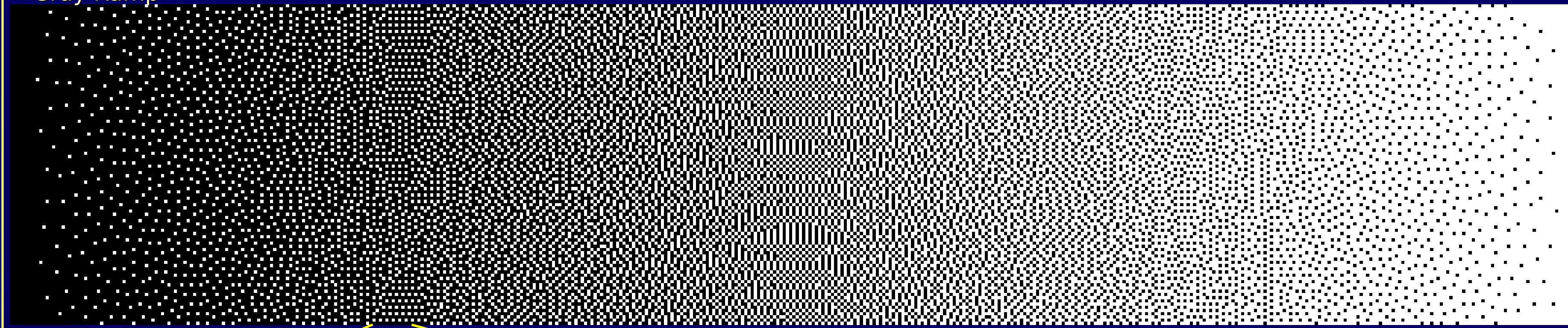
- **Compare Input(x,y) with Threshold(x,y)=.5**

- **Distribute Quantization Error on Unprocessed Pixels**

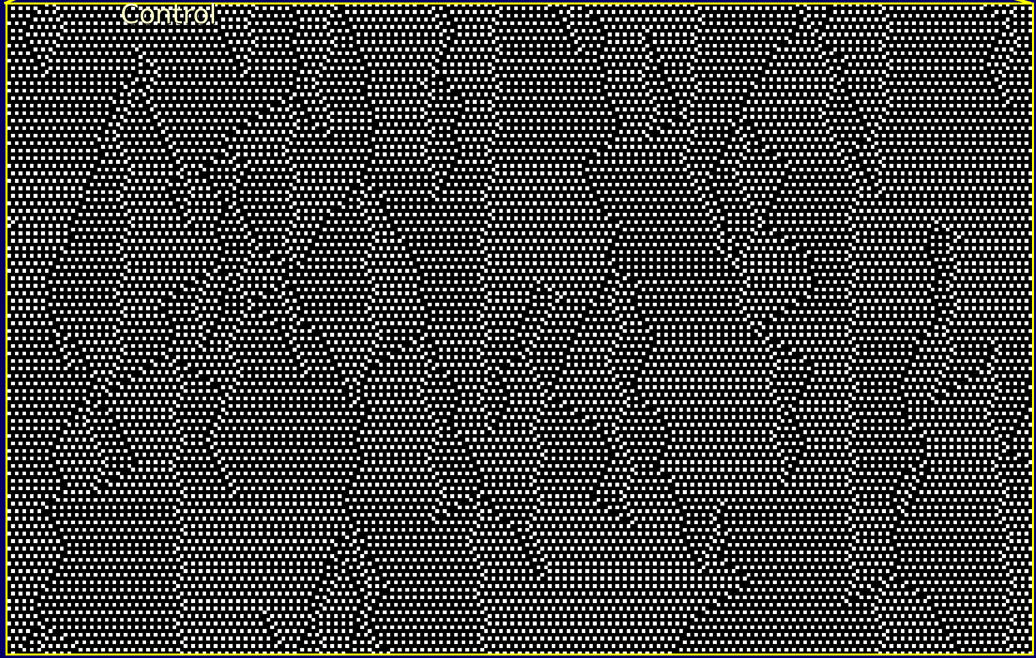


Floyd-Steinberg Error-Diffusion Algorithm

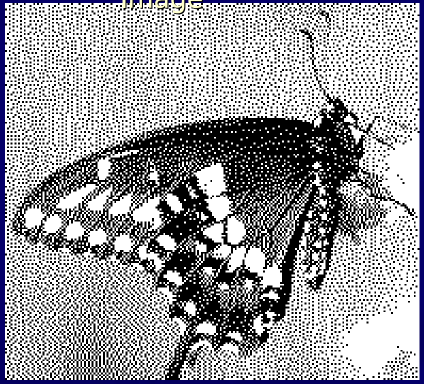
Gray Ramp



Intolerable Artifacts at Certain Intensity Levels, Out of Control

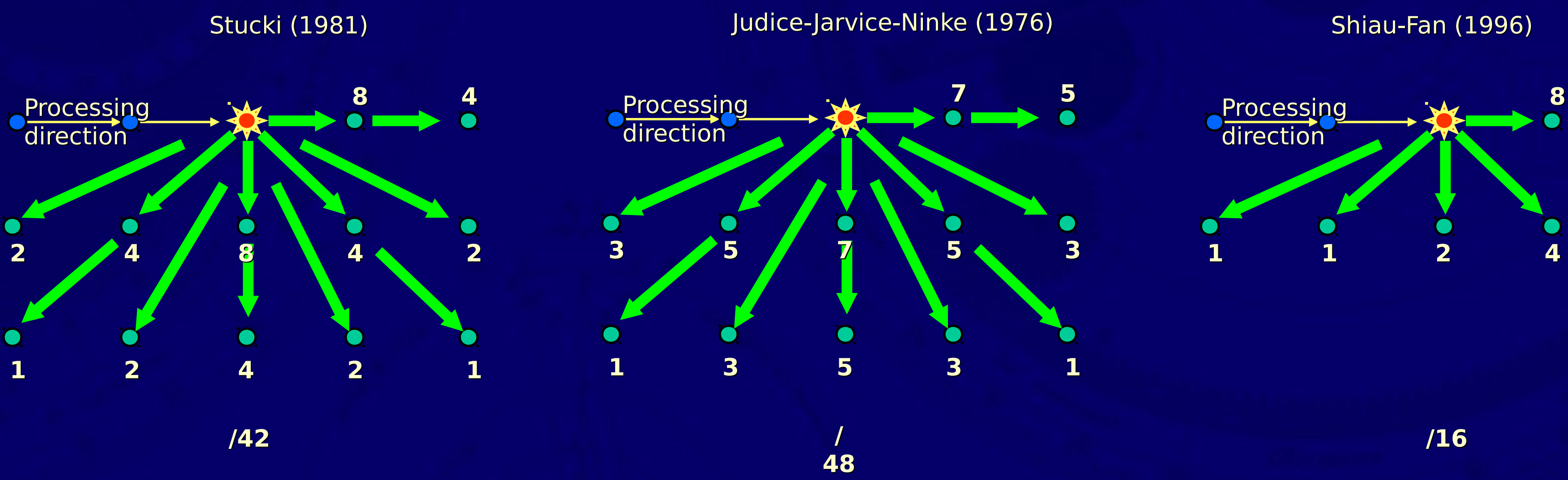


Sample Image



Sophistication of Error-Diffusion Algorithms

- Processing Paths: Serpentine, Space-Filling Curves
- Different Error Distribution Coefficients
- Threshold Modulation: Control of Edge Enhancement



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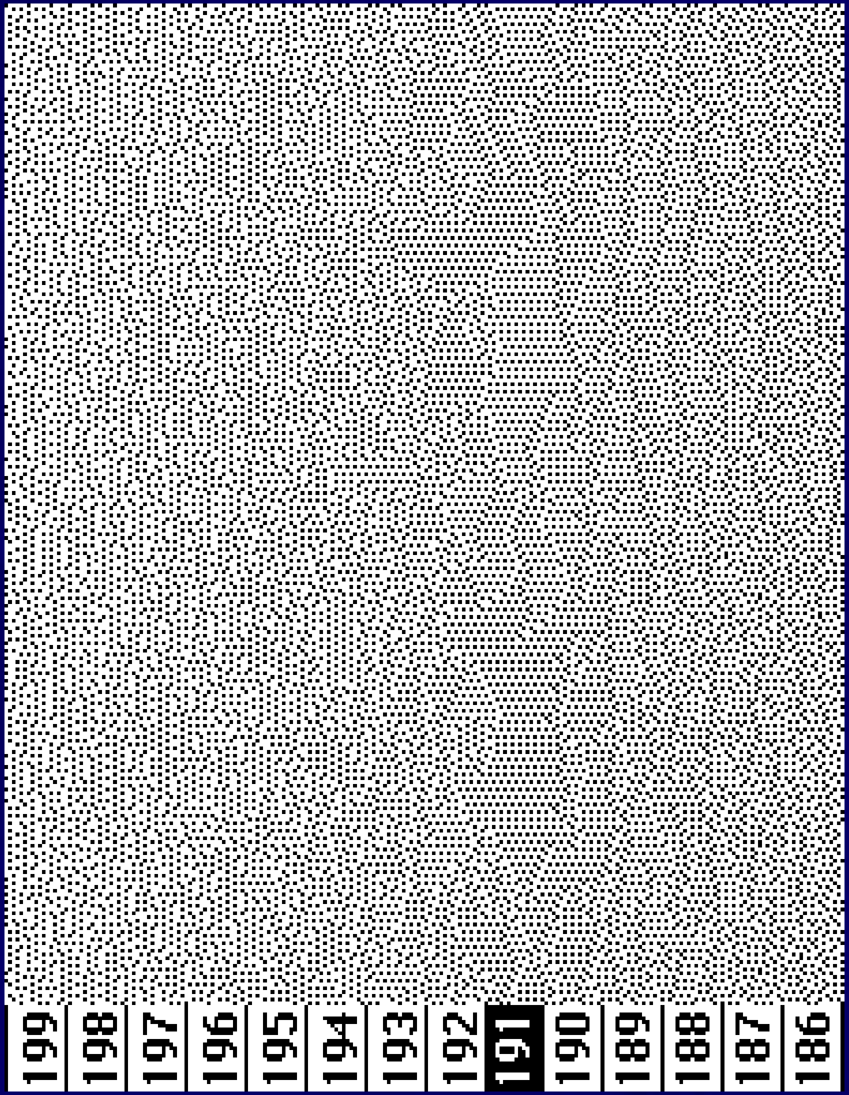
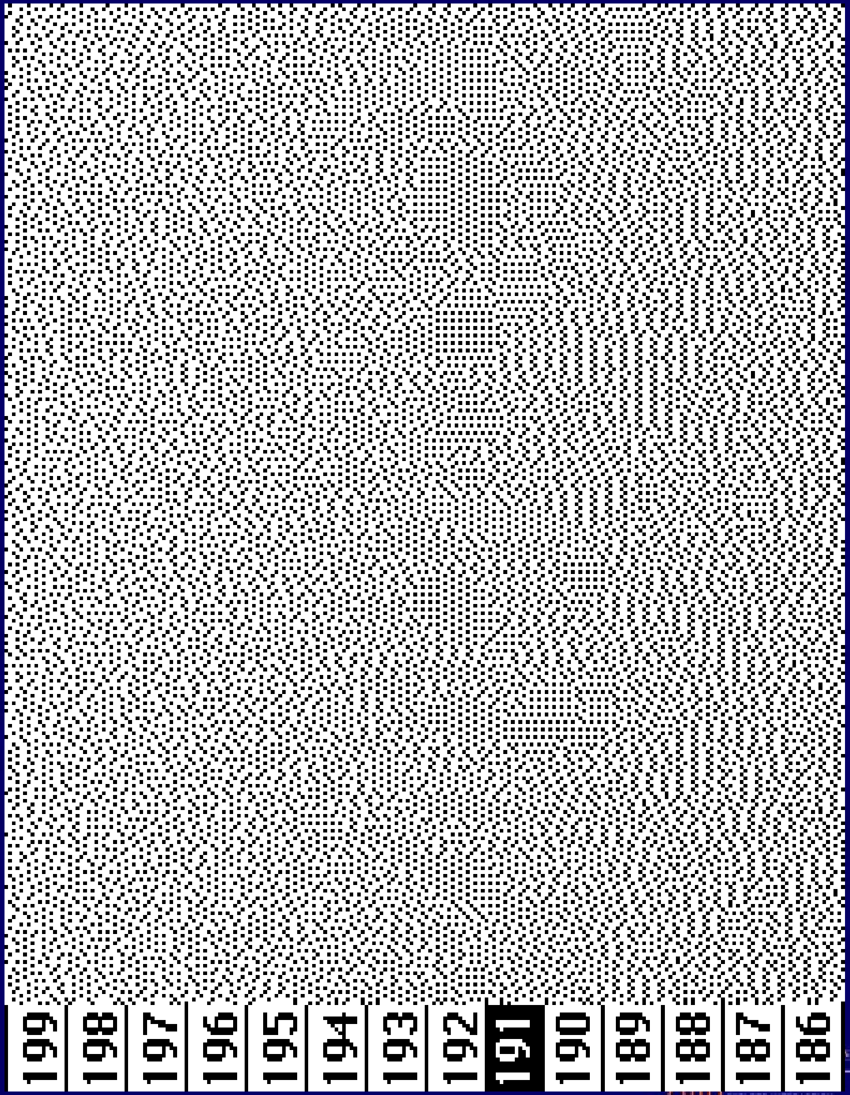
Hidden Artifacts

Std. Floyd-Steinberg E-D

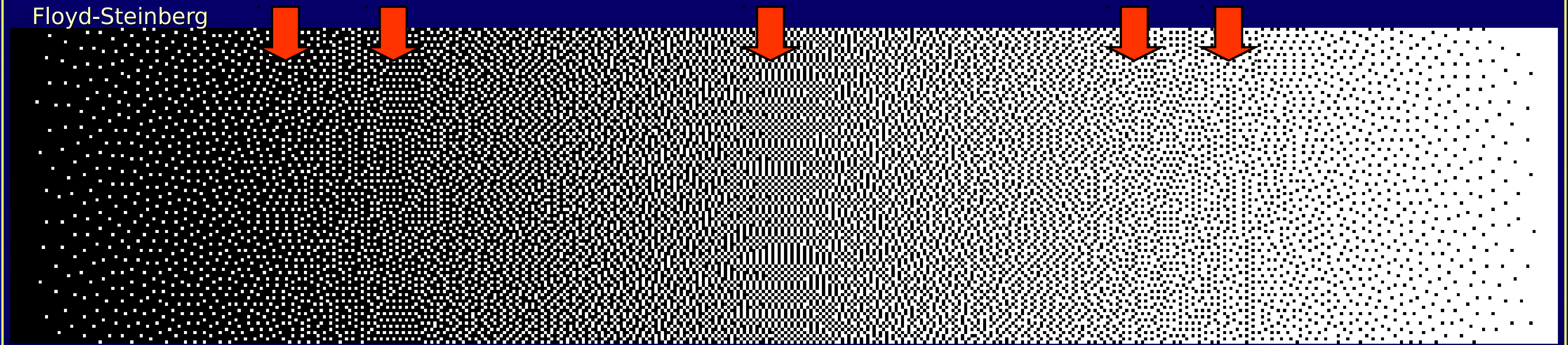


Shiau-Fan E-D



Different Error-Diffusion Algorithms

Floyd-Steinberg



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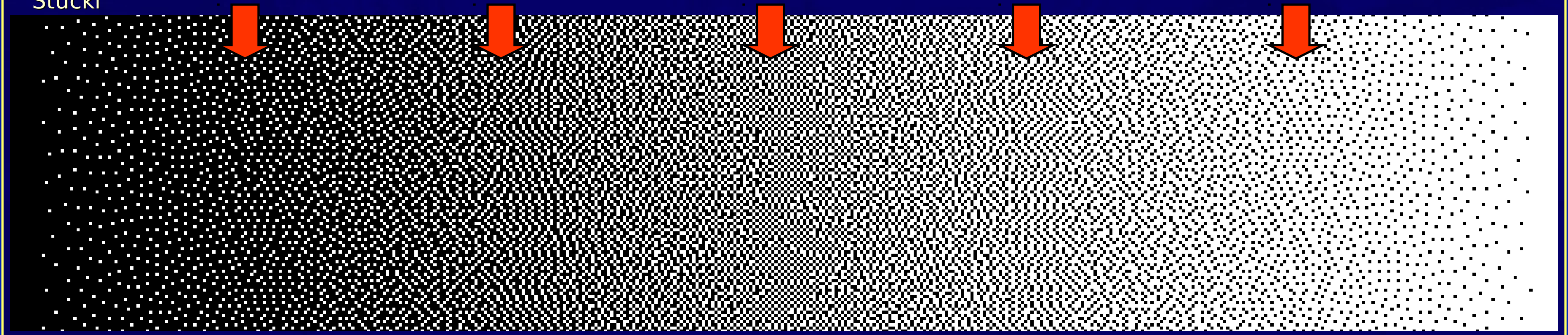
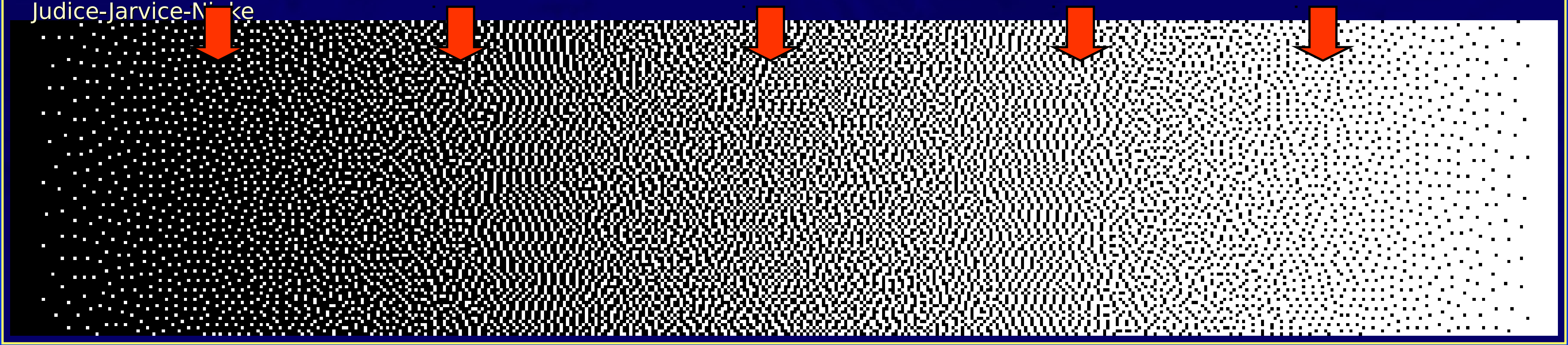
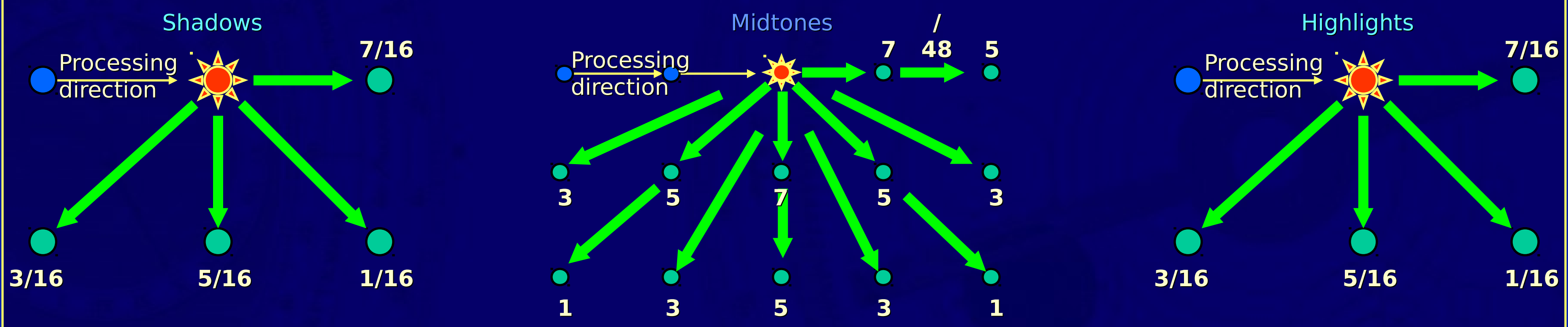
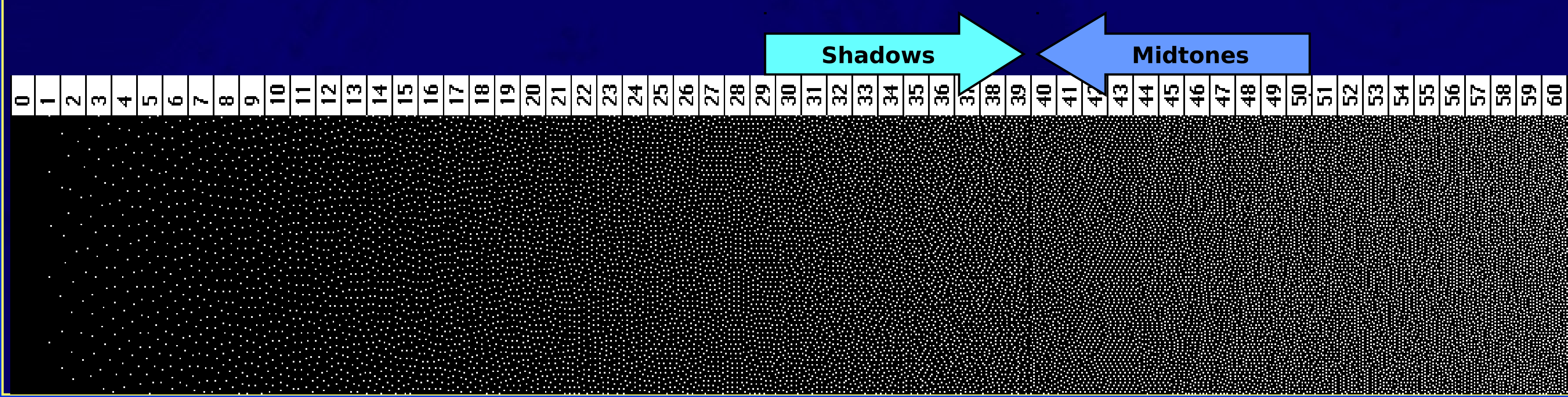


Judice-Jarvice-N[illegible]ke

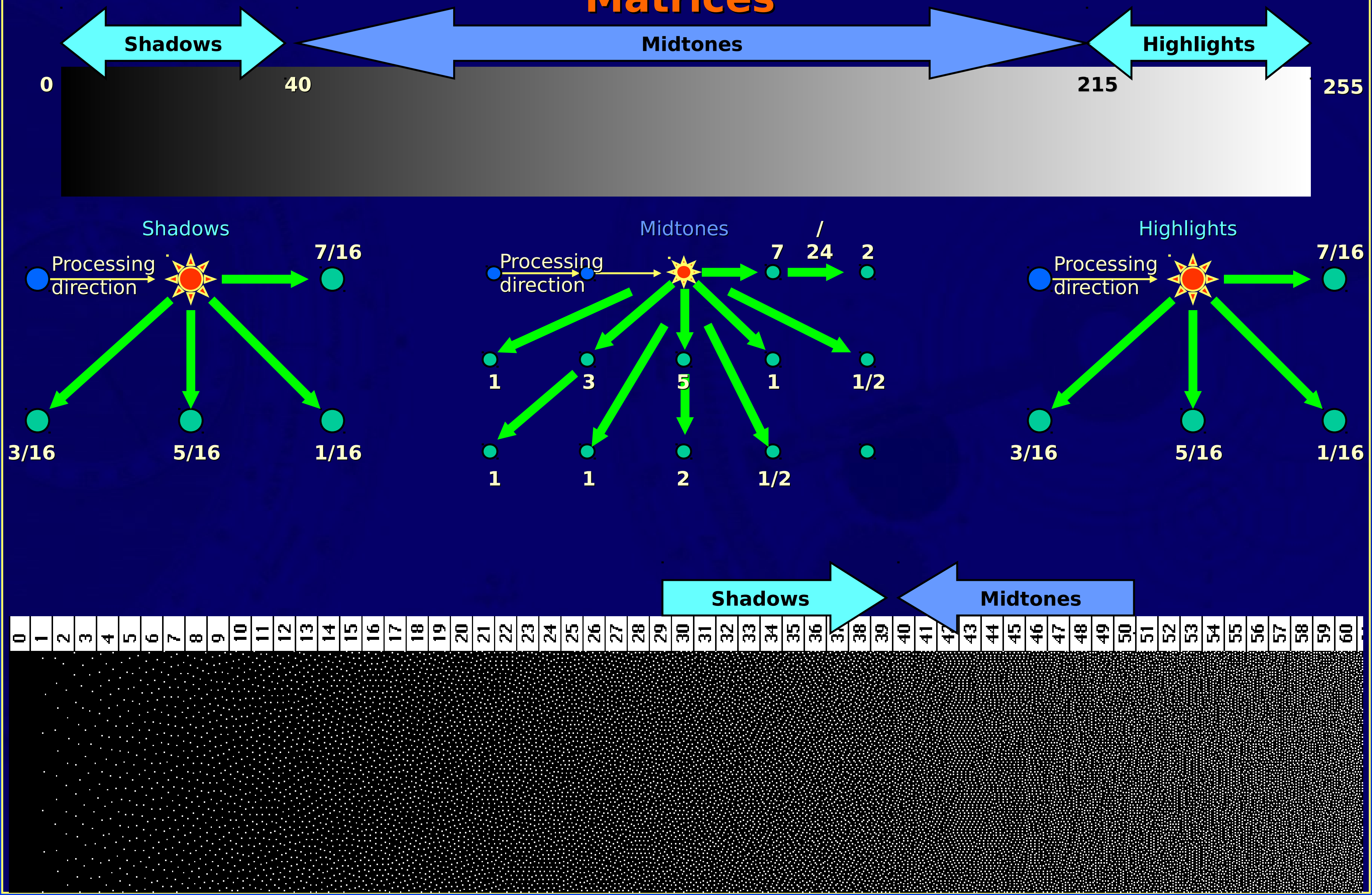


Eschbach: Input-Dependent Weights, Independent Matrices (1993)





Eschbach: Input-Dependent Weights, Dependent Matrices



Eschbach: Input-Dependent Weights

Conclusions

- Visual Artifacts can be improved Using Input-Dependent Weights
- Correlation Between Matrices May Reduce Banding Effect

Weak Points of the Method

- No Objective Criterion on Input-Dependent Weights
- No Mechanism Proposed To Insure Correlation Between Weights

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Our Contribution

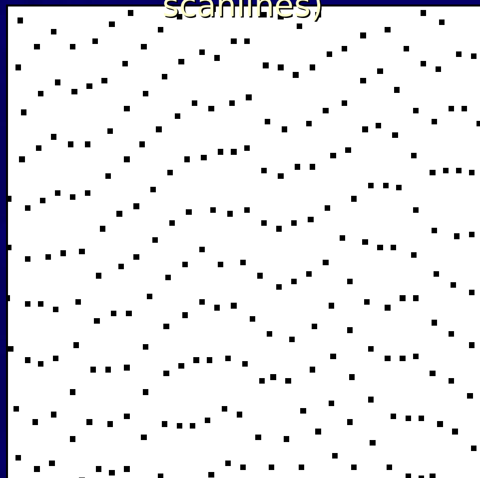
1. Provide Objective Criterion on Input-Dependent Weights
2. Provide Mechanism To Insure Correlation Between Weights
3. Implement an Error-Diffusion Algorithm
   - Conceptually Simple
   - Computationally Efficient
   - Almost Artifact-Free
   - Publicly Available

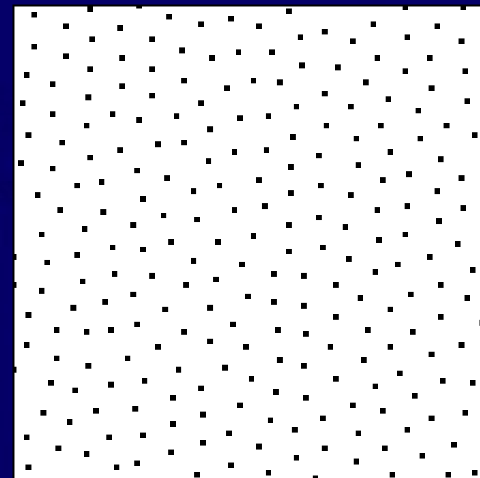
SIGGRAPH 2001 EXPLORE INTERACTION AND DIGITAL IMAGES

(1) Objective Criterion of Visual Quality of Halftoning

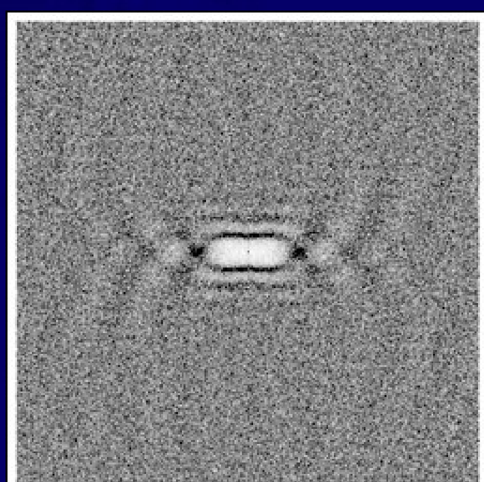
Bad Error-Diffusion
(Std. Floyd-Steinberg, scanlines)

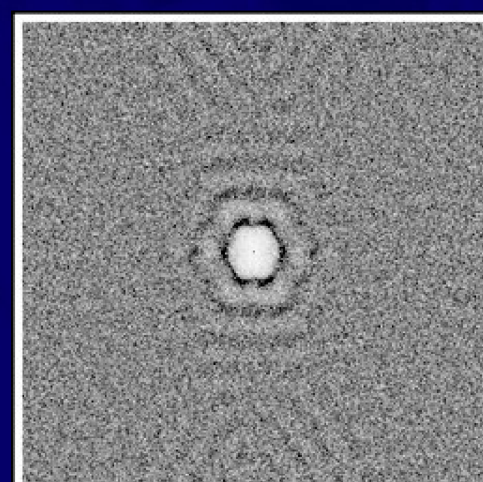
Good Error-Diffusion





Images

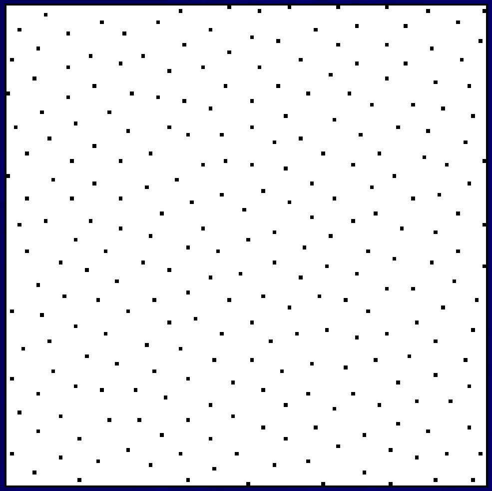




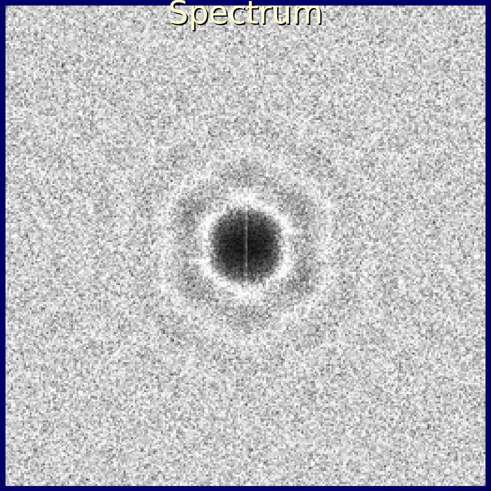
Fourier
Amplitude Spectra

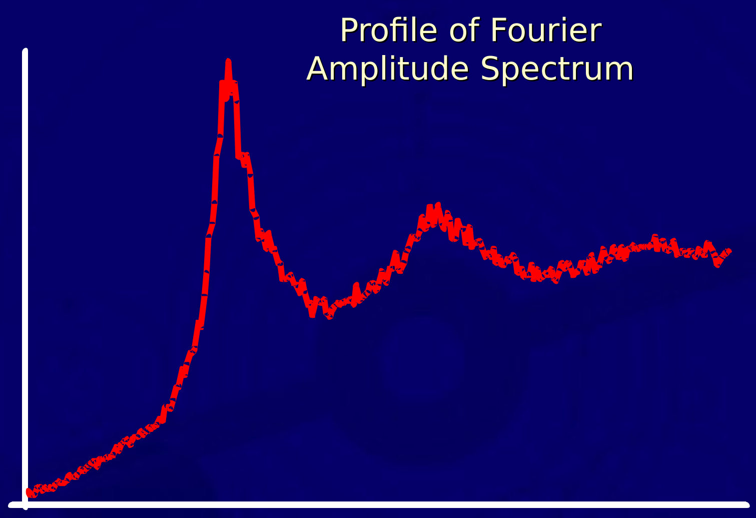
(1) Objective Criterion of Visual Quality of Halftoning

Image

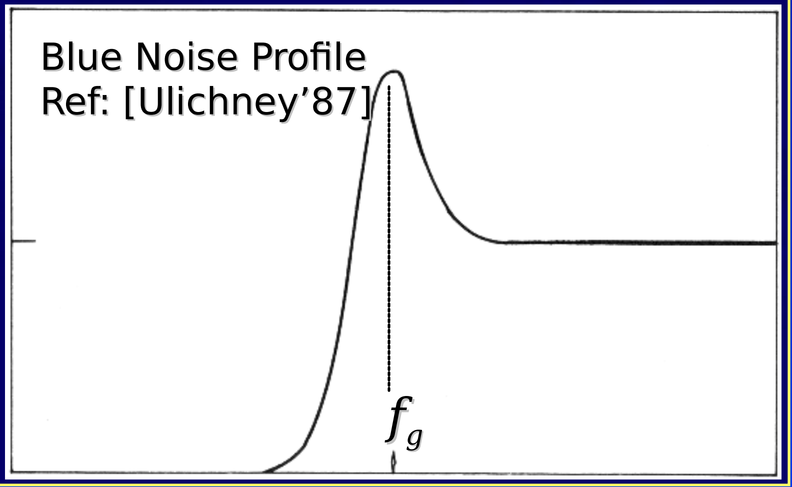


Fourier Amplitude Spectrum

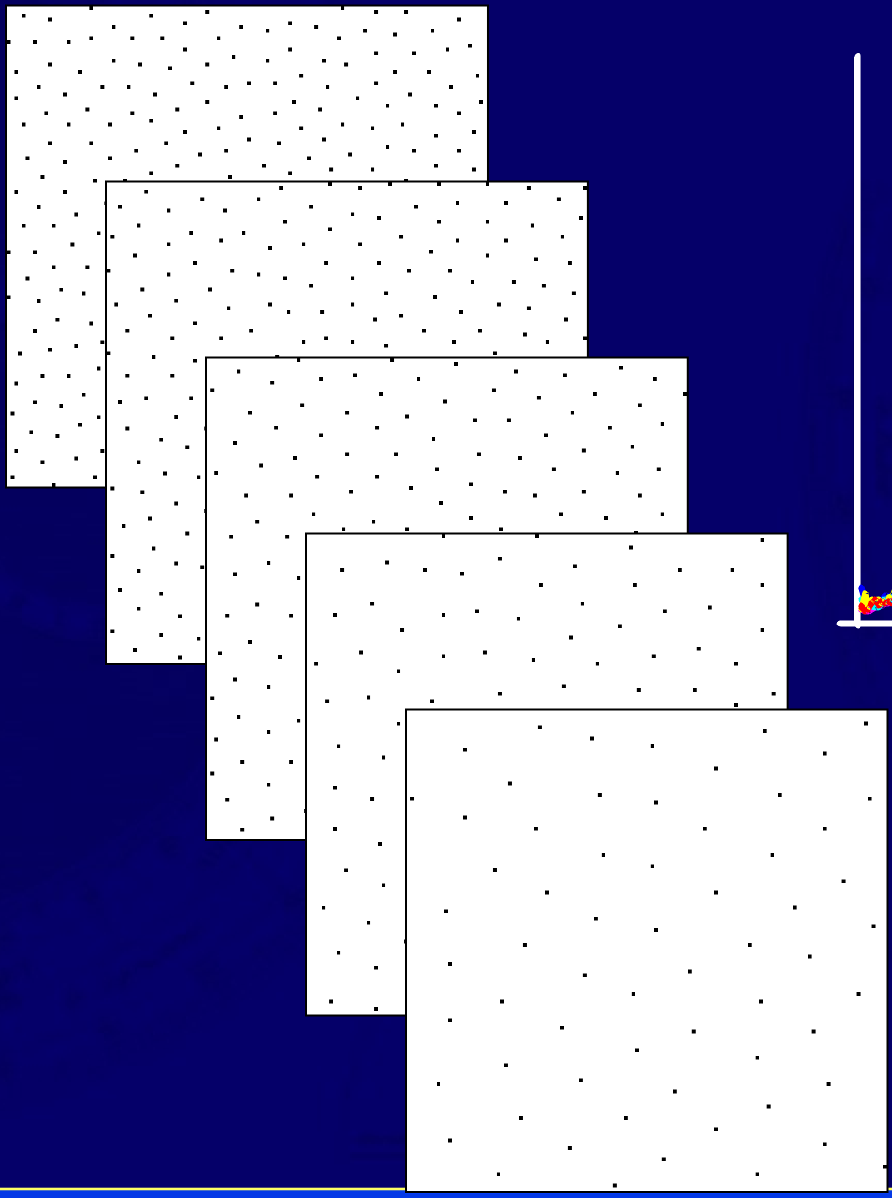


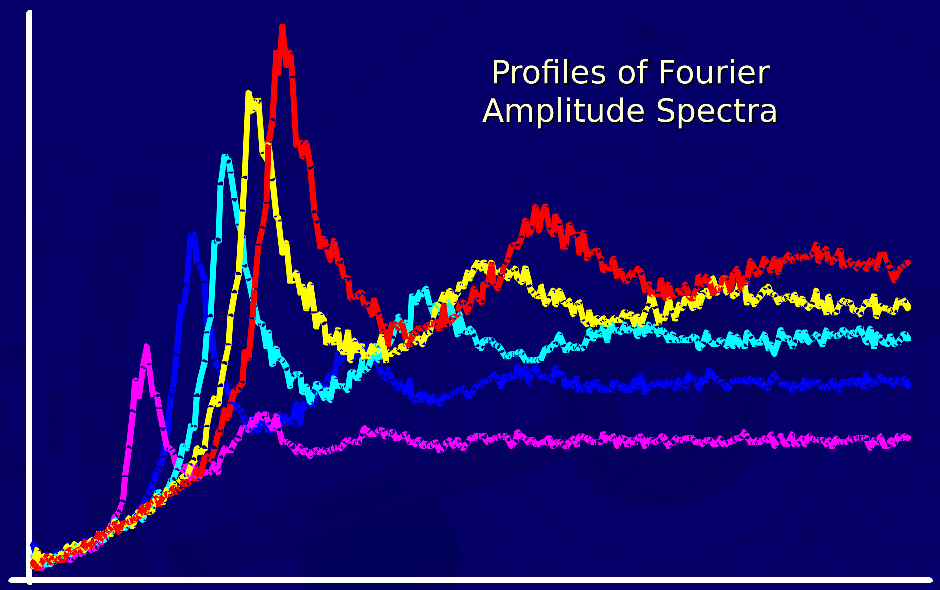


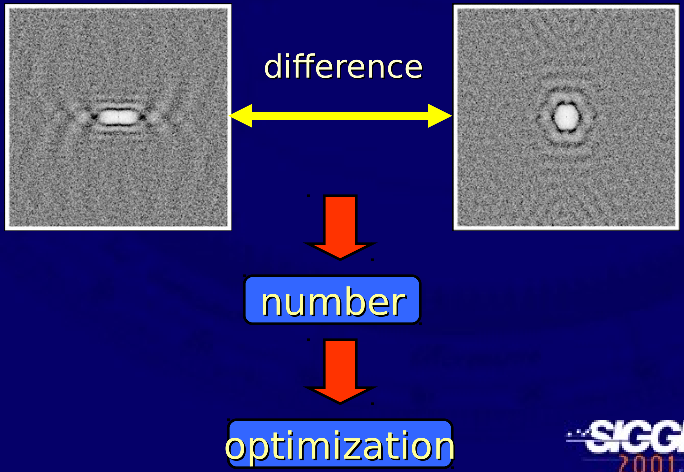
- **Radial Symmetry**
- **Characteristic Blue Noise Profile**



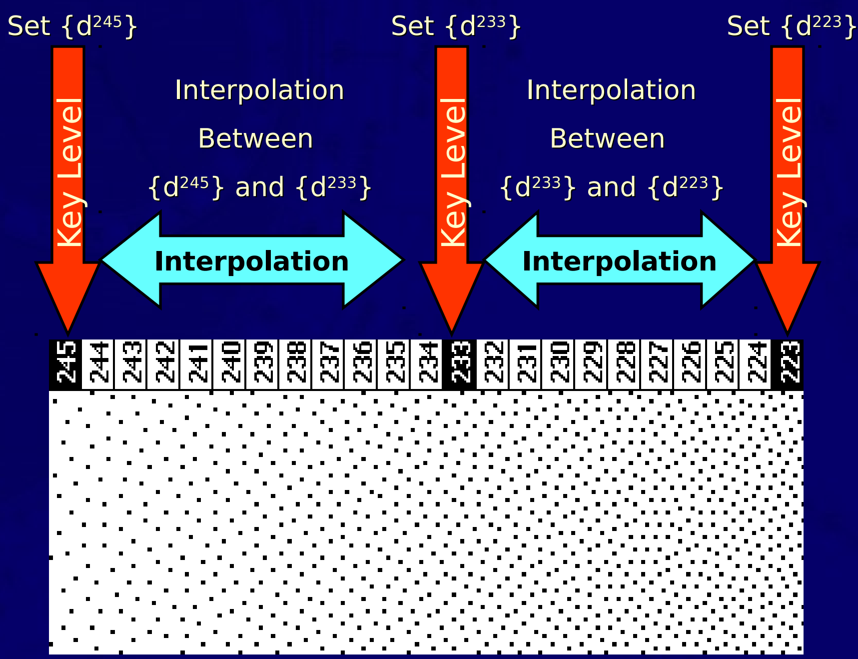
(1) Objective Criterion of Visual Quality of Halftoning



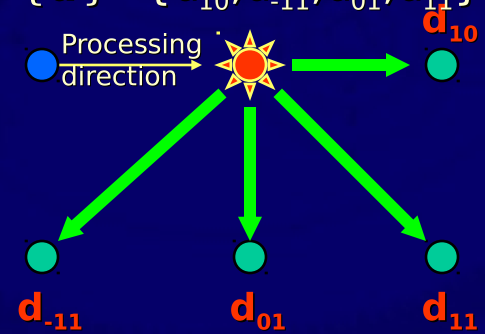




(2) Mechanism of Correlation Between E-D Weights



For each intensity level *i*,
The set $\{d^i\}$ is defined
as
$\{d^i\}=\{d_{10},d_{-11},d_{01},d_{11}\}$



Our Error-Diffusion Algorithm

Assumptions:

A. E-D Algorithm is good when its Fourier Amplitude Spectrum is close to “Blue Noise”

B. Smooth behavior of E-D algorithm when coefficients $\{d_i\}$ vary smoothly across the dynamic range of intensity levels $\{i\}$

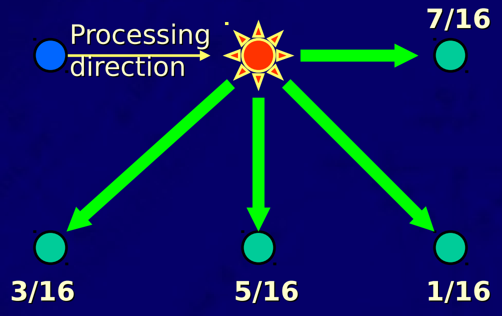
C. Intensity levels 0, **1/4, 1/3, 1/2, 2/3, 3/4, 1** are potentially problematic: they may be source of artifacts

D. Sets $\{d_i\}$ and $\{d_{255-i}\}$ generate almost identical artifacts structures, with white and black inverted

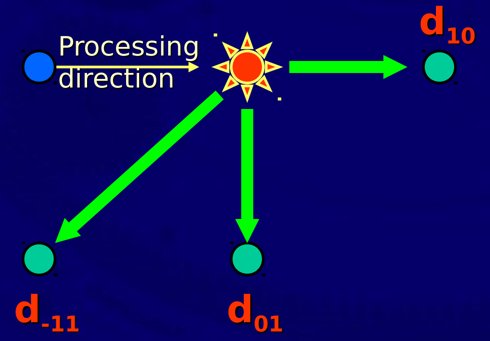
Our Error-Diffusion Algorithm

- **Simplified Set of Distribution Coefficients** $d_i=\{d_{10},d_{-11},d_{01}\}$ for Each Intensity Level i
  - **Faster**
  - **Easier to Optimize**
  - **Sufficient for Achieving Good Quality**

Floyd-Steinberg

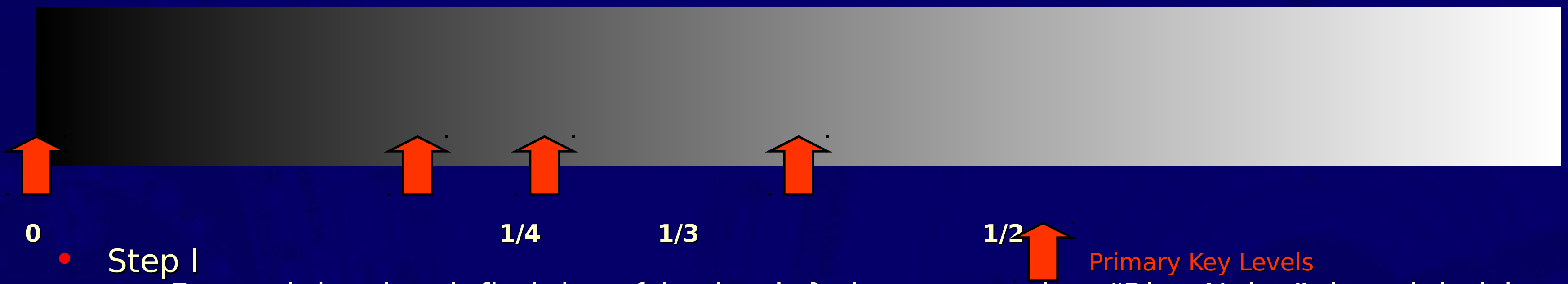


Our Algorithm



Our Error-Diffusion Algorithm: Finding $\{d_{10}, d_{-11}, d_{01}\}$

Given: Initial set of key level between 0 and ½, where artifacts are *a priori* present



- Step I
  For each key level, find $d^{key} = \{d_{10}, d_{-11}, d_{01}\}$ that approaches "Blue Noise", by minimizing the difference between given Fourier Amplitude Spectrum and Ideal "Blue Noise Profile"

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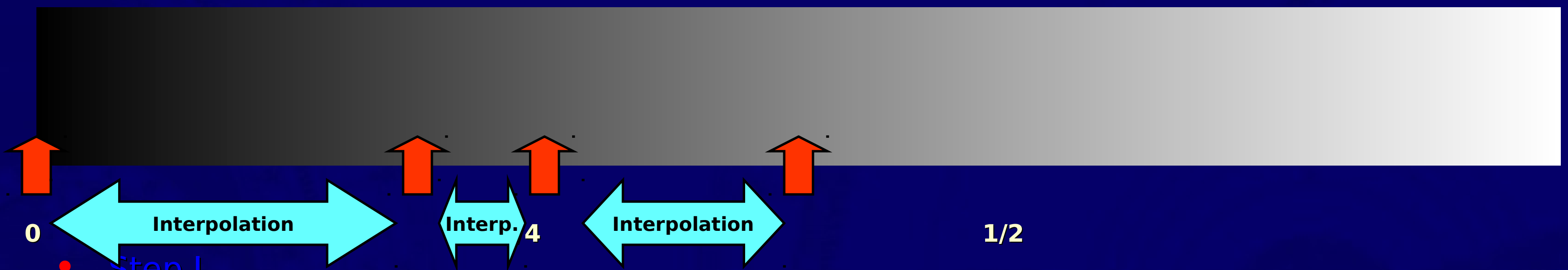

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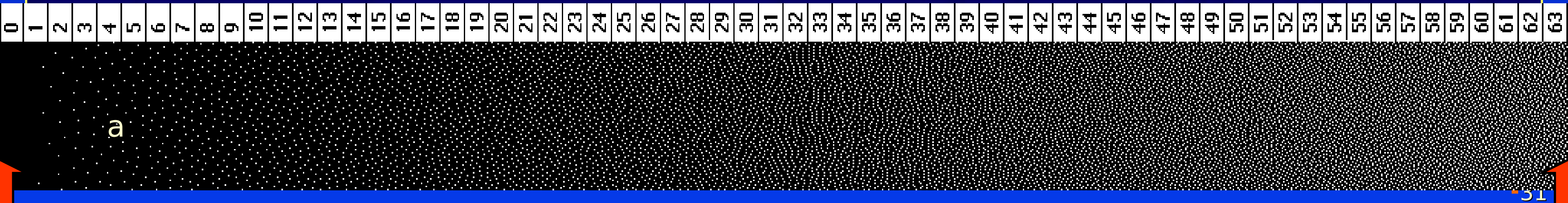
- Step II
  - Interpolate between key level.
  - Visually check for artifacts in-between.



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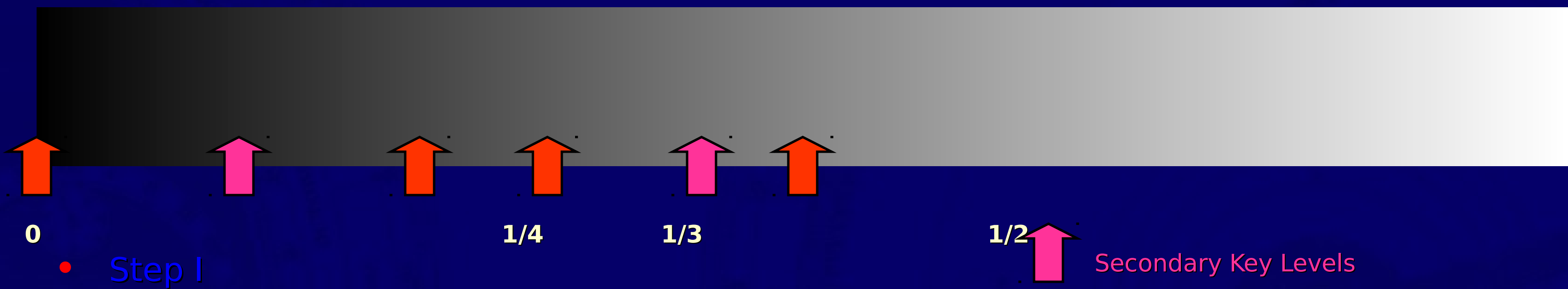

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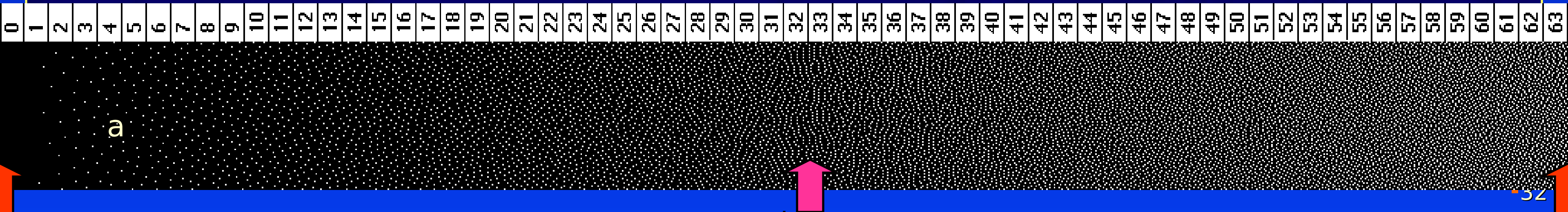
  If not OK,
  define new key levels and jump to Step I
  otherwise:
  continue



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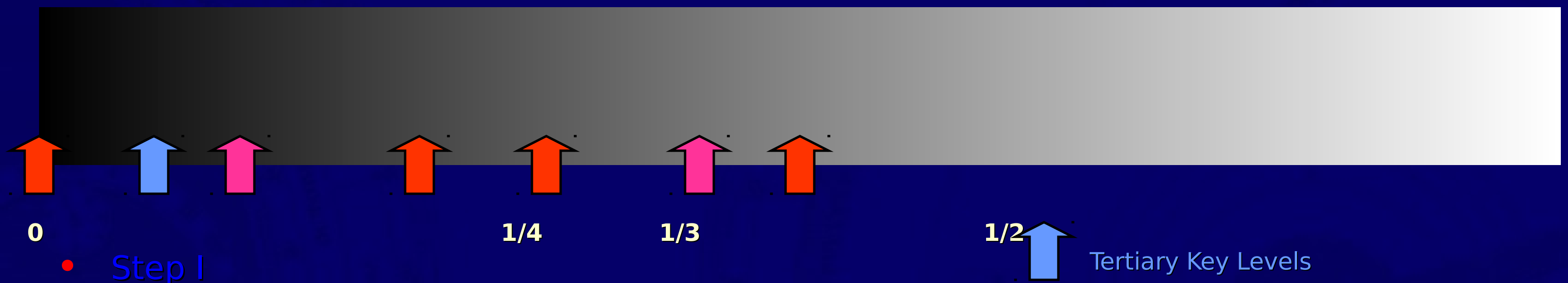

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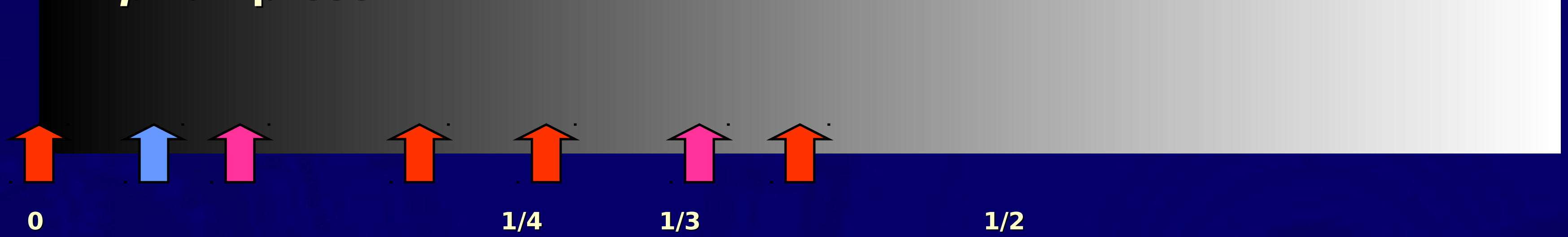

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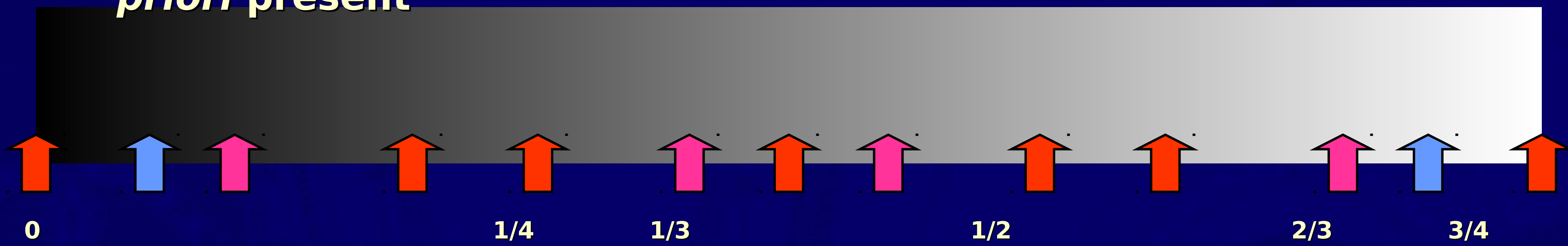
- Step III
  Extend Solution Symmetrically, about ½

SIGGRAPH 2001

Our Error-Diffusion Algorithm: Finding $\{d_{10},d_{-11},d_{01}\}$

Given: Initial set of key level between 0 and ½, where artifacts are *a priori* present



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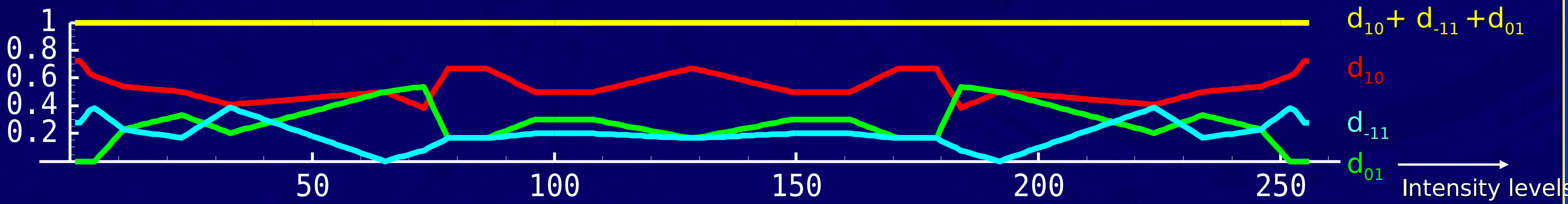
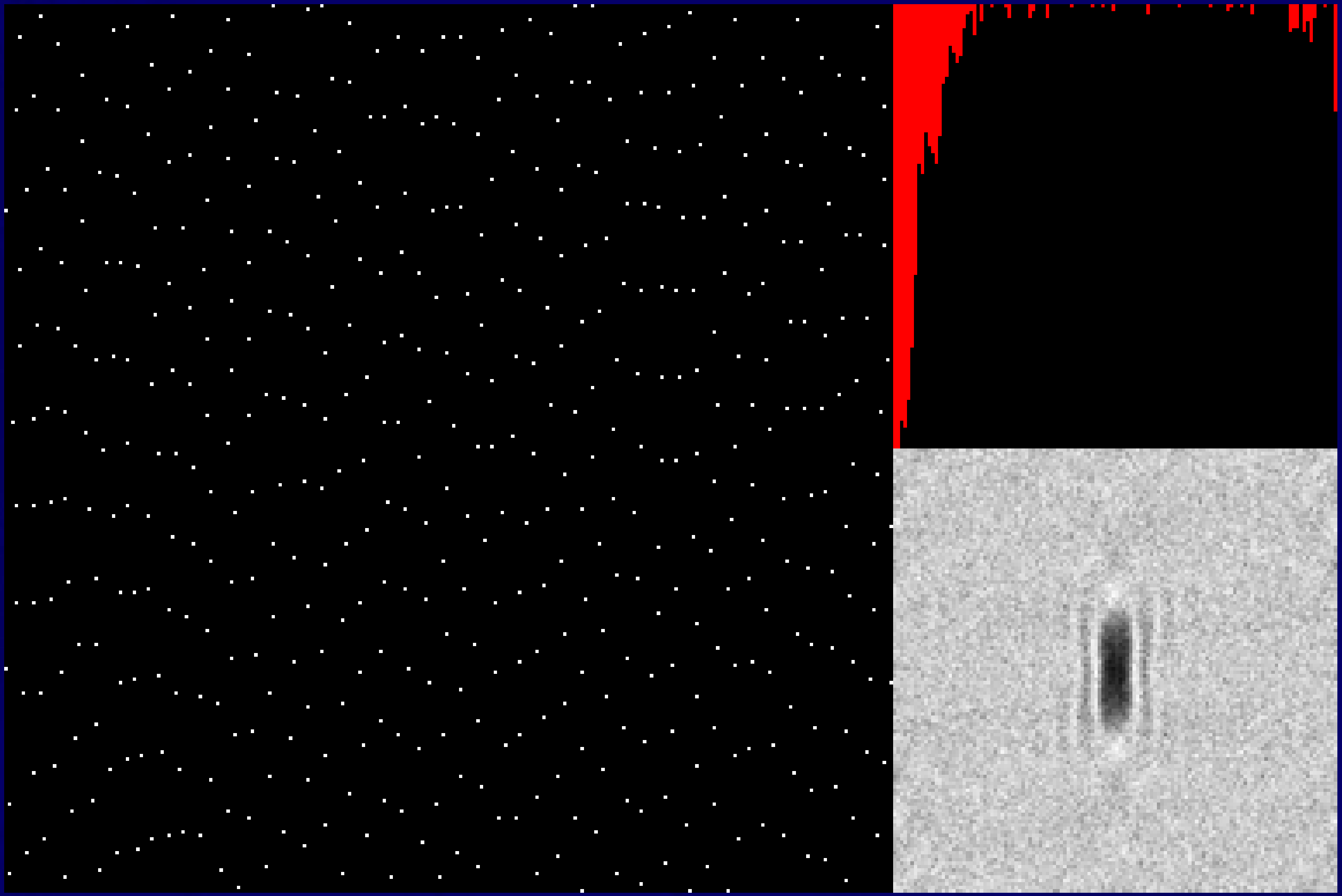
Finding $\{d_{10}, d_{-11}, d_{01}\}$: Results

0:	13, 0, 5	**32:**	20, 10, 19	**64:**	11, 10, 0	96:	5, 3, 2
1:	13, 0, 5	33:	1937,1000,1767	65:	158, 151, 3	97:	5, 3, 2
2:	21, 0, 10	34:	977, 520, 855	66:	178, 179, 7	98:	5, 3, 2
3:	7, 0, 4	35:	657, 360, 551	67:	1030,1091,63	99:	5, 3, 2
4:	8, 0, 5	36:	71, 40, 57	68:	248, 277, 21	100:	5, 3, 2
5:	47, 3, 28	37:	2005,1160,1539	69:	318, 375, 35	101:	5, 3, 2
6:	23, 3, 13	38:	337, 200, 247	70:	458, 571, 63	102:	5, 3, 2
7:	15, 3, 8	39:	2039,1240,1425	71:	878, 1159,147	103:	5, 3, 2
8:	22, 6, 11	40:	257, 160, 171	**72:**	5, 7, 1	104:	5, 3, 2
9:	43, 15, 20	41:	691, 440, 437	73:	172, 181, 37	105:	5, 3, 2
10:	7, 3, 3	42:	1045,680, 627	74:	97, 76, 22	106:	5, 3, 2
11:	501, 224, 211	43:	301, 200, 171	75:	72, 41, 17	**107:**	5, 3, 2
12:	249, 116, 103	44:	177, 120, 95	76:	119, 47, 29	108:	305, 176, 119
13:	165, 80, 67	45:	2141,1480,1083	**77:**	4, 1, 1	109:	155, 86, 59
14:	123, 62, 49	46:	1079,760, 513	78:	4, 1, 1	110:	105, 56, 39
15:	489, 256, 191	47:	725, 520, 323	79:	4, 1, 1	111:	80, 41, 29
16:	81, 44, 31	48:	137, 100, 57	80:	4, 1, 1	112:	65, 32, 23
17:	483, 272, 181	49:	2209,1640,855	81:	4, 1, 1	113:	55, 26, 19
18:	60, 35, 22	50:	53, 40, 19	82:	4, 1, 1	114:	335, 152, 113
19:	53, 32, 19	51:	2243,1720,741	83:	4, 1, 1	115:	85, 37, 28
20:	237, 148, 83	52:	565, 440, 171	84:	4, 1, 1	116:	115, 48, 37
21:	471, 304, 161	53:	759, 600, 209	**85:**	4, 1, 1	117:	35, 14, 11
22:	3, 2, 1	54:	1147,920, 285	86:	65, 18, 17	118:	355, 136, 109
23:	481, 314, 185	55:	2311,1880,513	87:	95, 29, 26	119:	30, 11, 9
24:	354, 226, 155	56:	97, 80, 19	88:	185, 62, 53	120:	365, 128, 107
25:	1389,866, 685	57:	335, 280, 57	89:	30, 11, 9	121:	185, 62, 53
26:	227, 138, 125	58:	1181,1000,171	90:	35, 14, 11	122:	25, 8, 7
27:	267, 158, 163	59:	793, 680, 95	91:	85, 37, 28	123:	95, 29, 26
28:	327, 188, 220	60:	599, 520, 57	92:	55, 26, 19	124:	385, 112, 103
29:	61, 34, 45	61:	2413,2120,171	93:	80, 41, 29	125:	65, 18, 17
30:	627, 338, 505	62:	405, 360, 19	94:	155, 86, 59	126:	395, 104, 101
31:	1227,638, 1075	63:	2447,2200,57	**95:**	5, 3, 2	**127:**	4, 1, 1

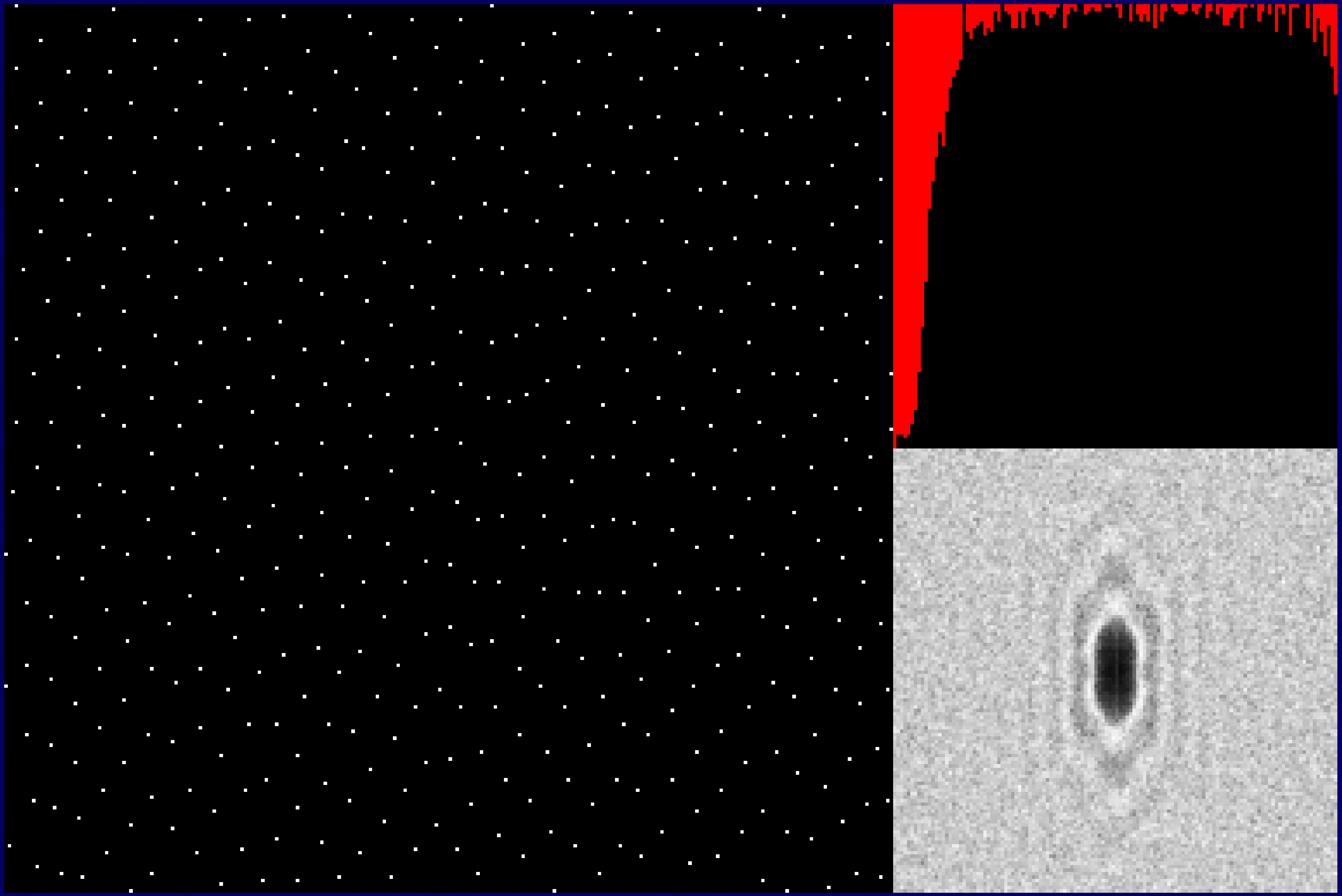


Step I, Iteration # 1



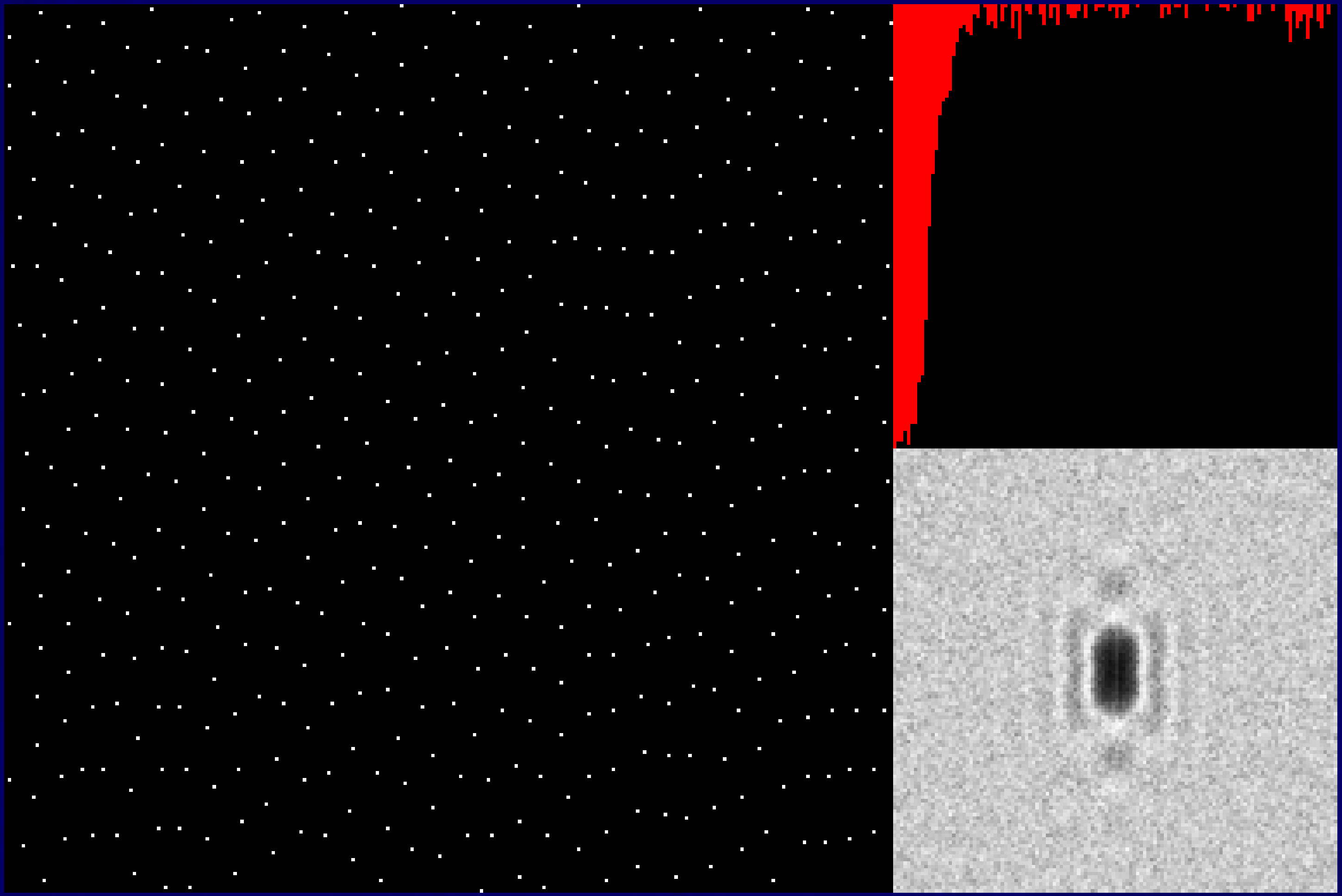
d_{10} = 0.333333
d_{-11} = 0.333333
d_{01} = 0.333333

Step I, Iteration # 3



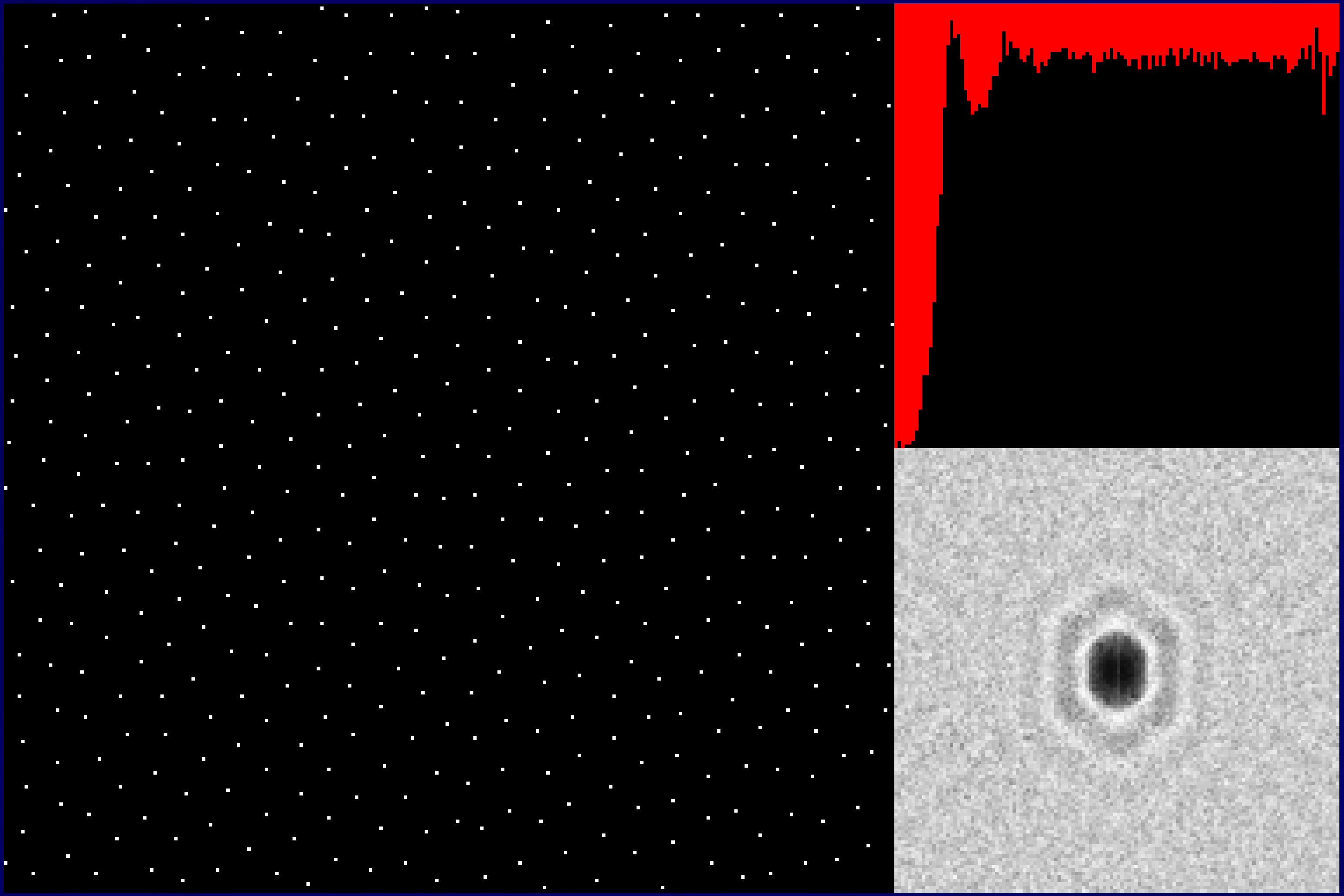
d_{10} = 0.445736
d_{-11} = 0.306202
d_{01} = 0.248062

Step I, Iteration # 7



d_{10} = 0.527778
d_{-11} = 0.444445
d_{01} = 0.0277776

Step I, Iteration # 12

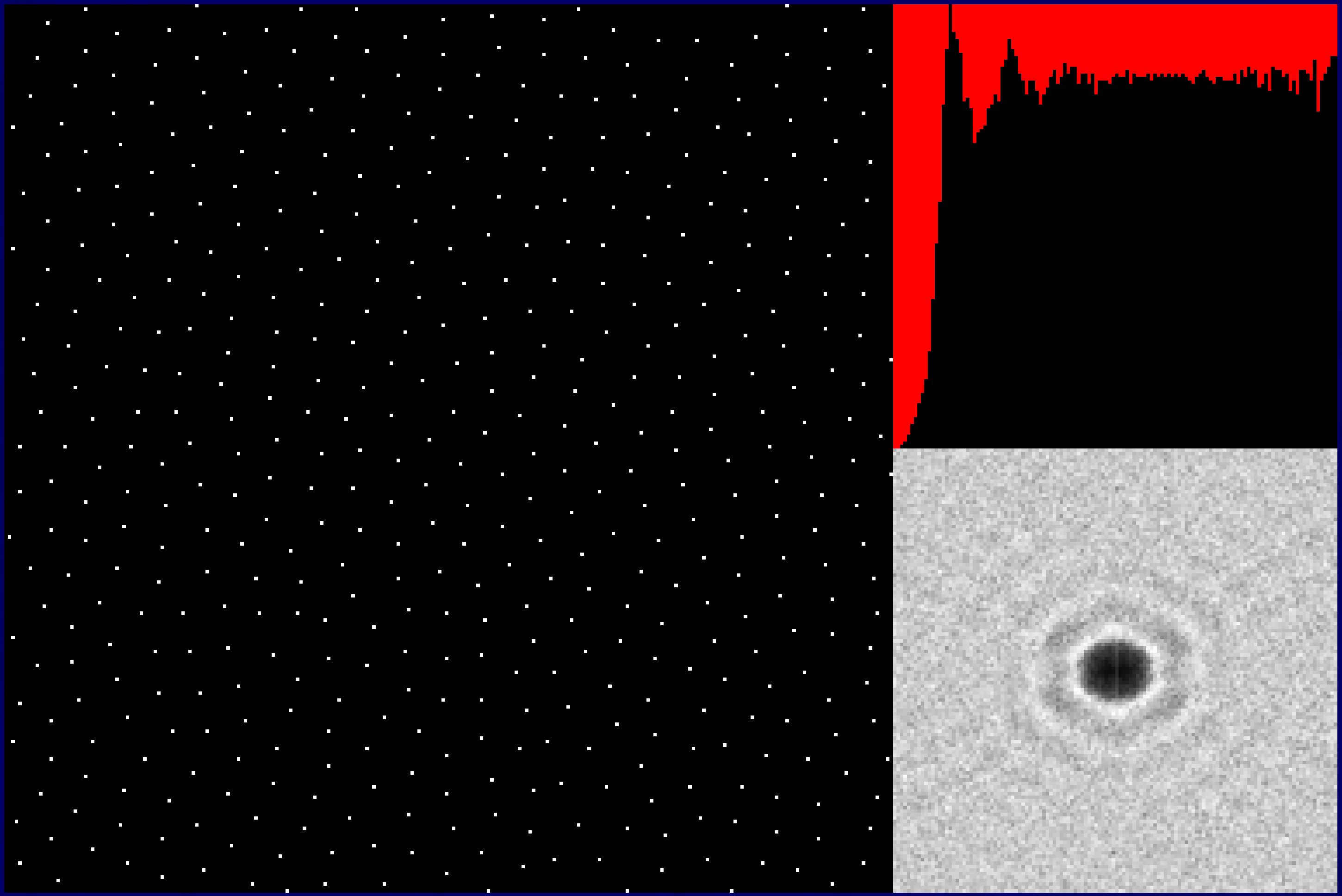


d_{10} = 0.637344

d_{-11} = 0.186967

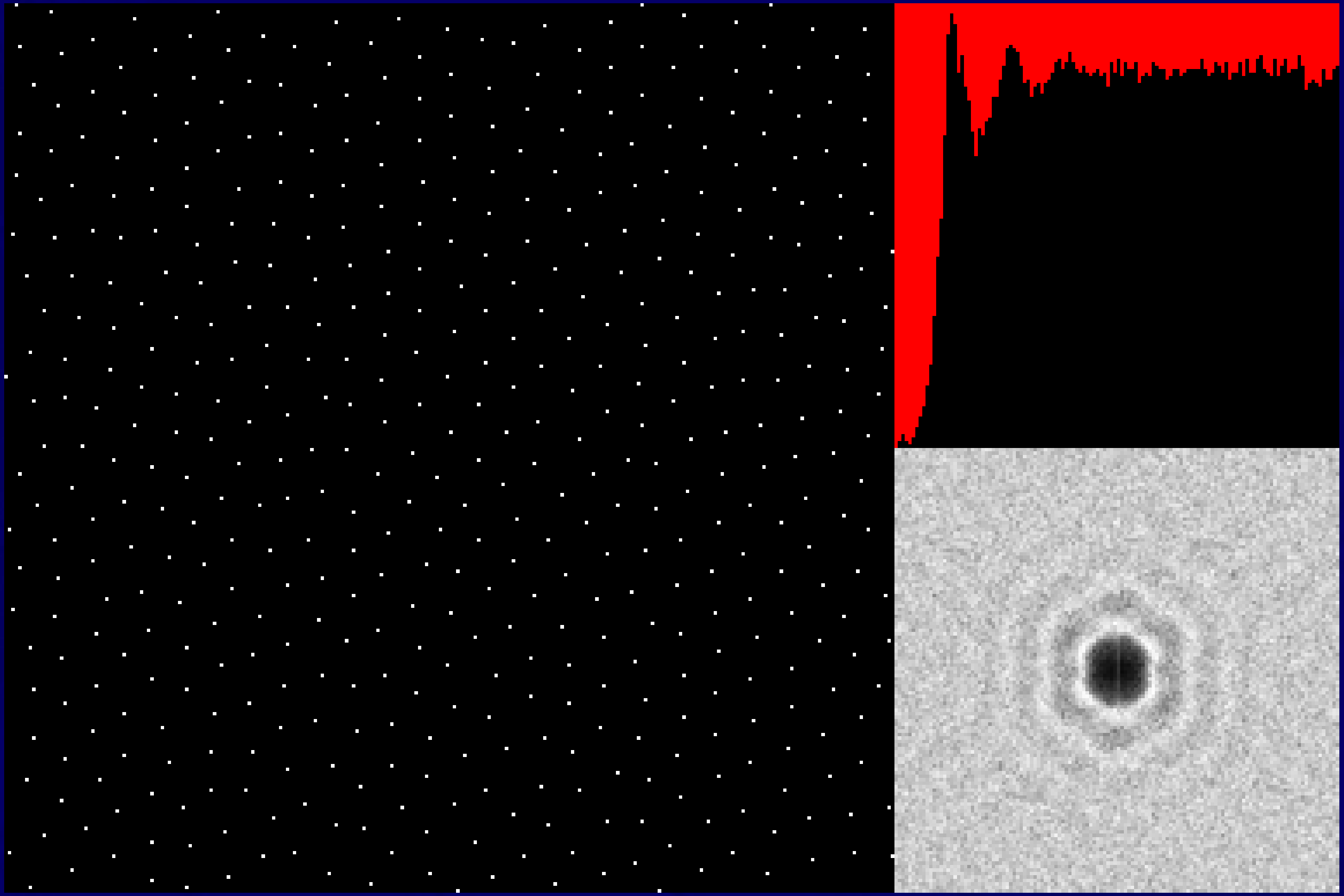
d_{01} = 0.175689

Step I, Iteration # 15



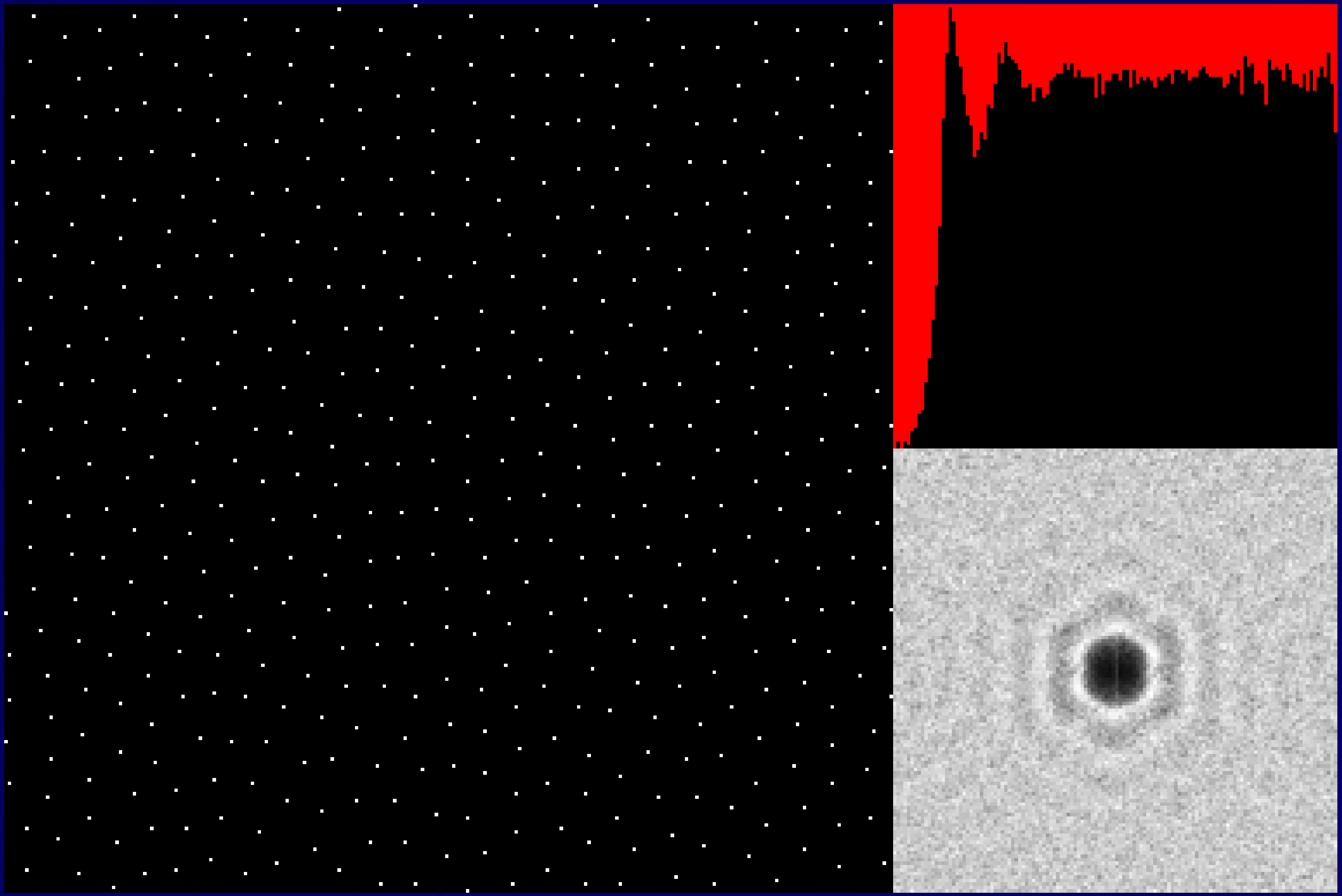
d_{10} = 0.630038
d_{-11} = 0.218744
d_{01} = 0.151219

Step I, Iteration # 20



d_{10} = 0.673123
d_{-11} = 0.114484
d_{01} = 0.212393

Step I, Iteration # 33



d_{10} = 0.664381
d_{-11} = 0.185621
d_{01} = 0.149998

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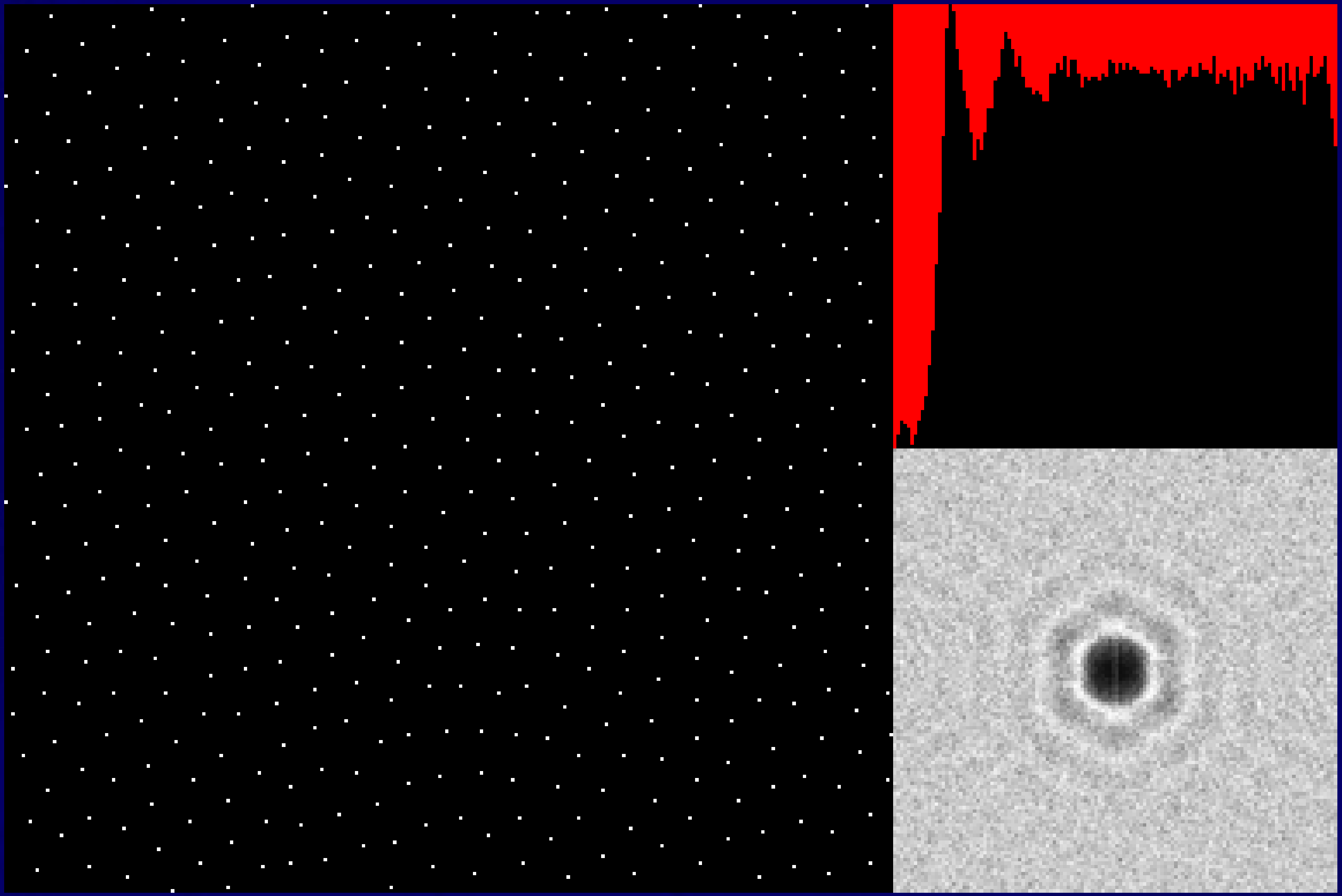
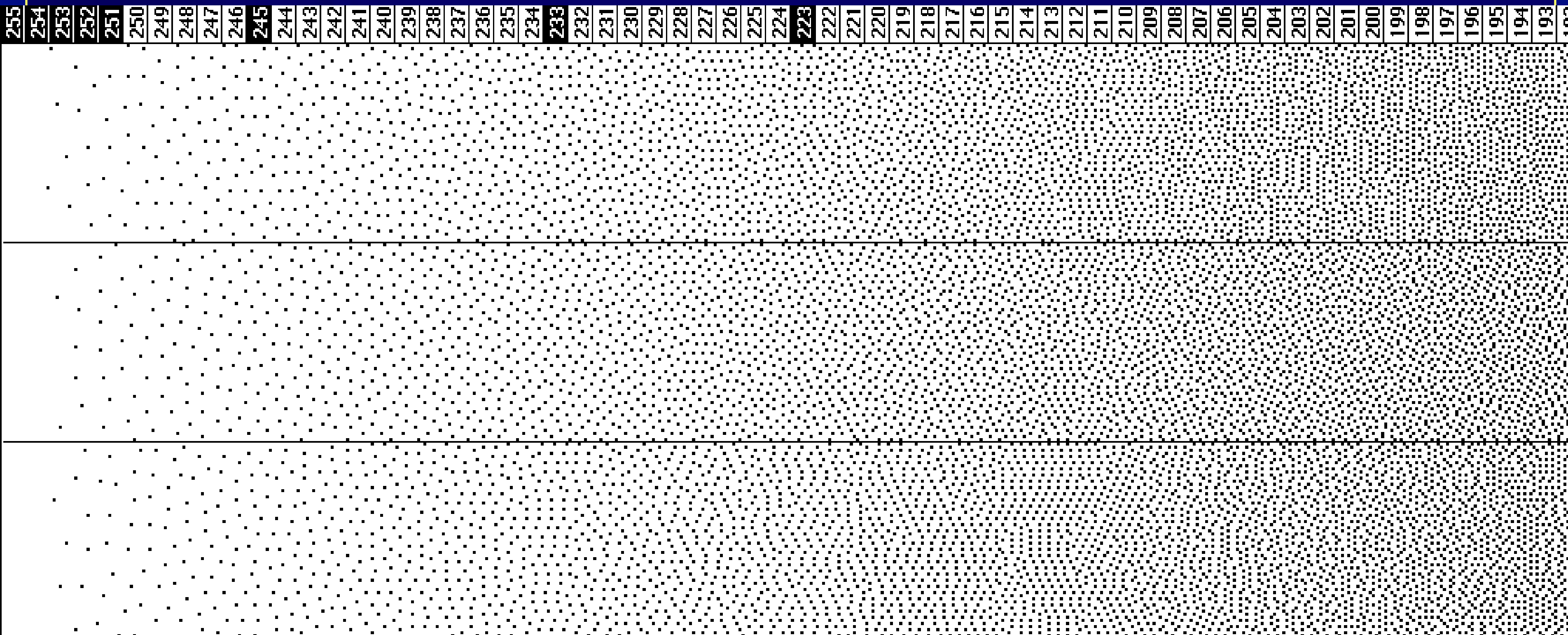
Step I, Iteration # 41



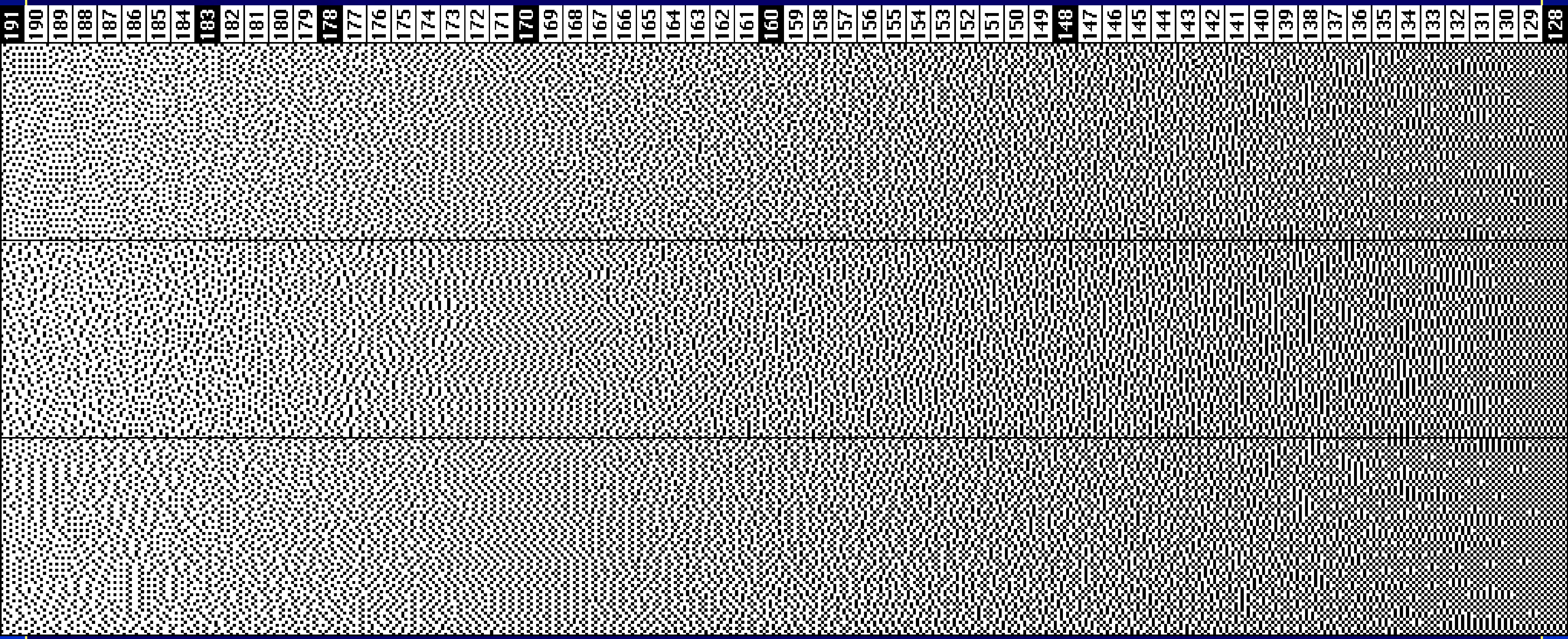
d_{10} = 0.655903
d_{-11} = 0.182528
d_{01} = 0.16157

Results



Top: Std. Floyd-Steinberg E-D
Middle: Our Method
Bottom: Shiau-Fan E-D

Results



Top: Std. Floyd-Steinberg E-D
Middle: Our Method
Bottom: Shiau-Fan E-D

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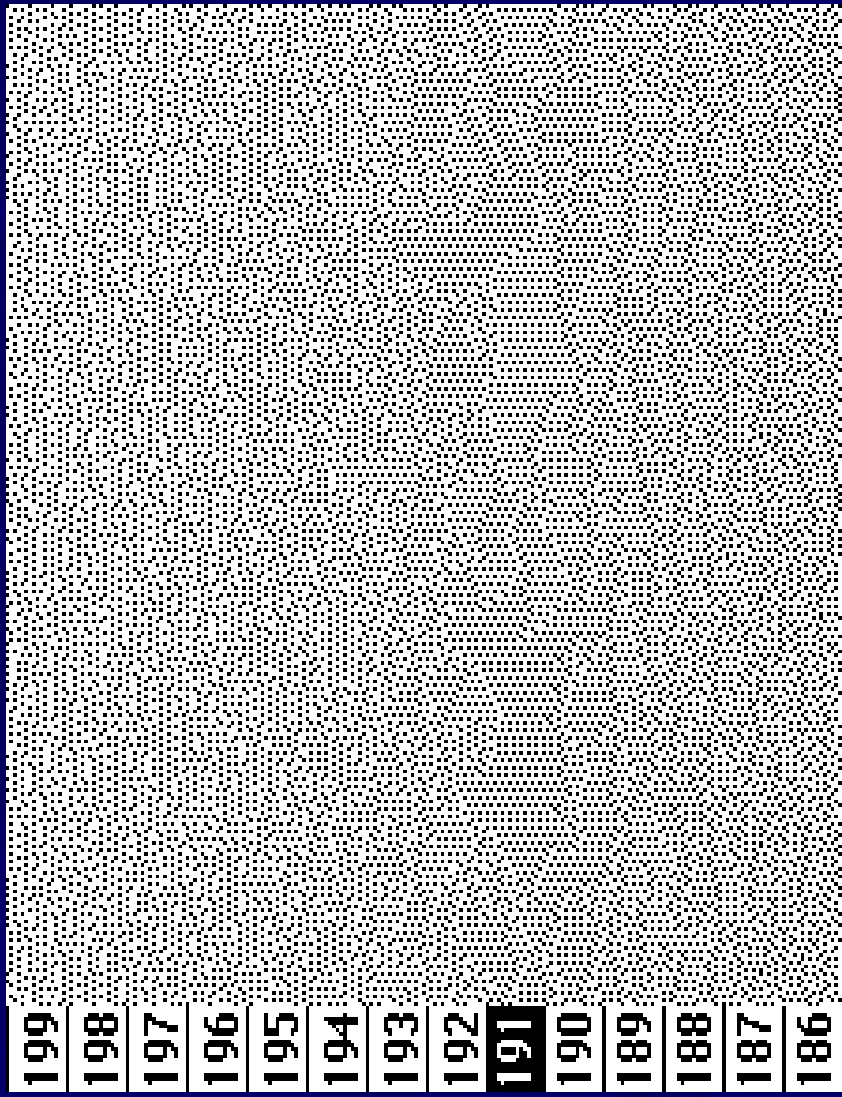
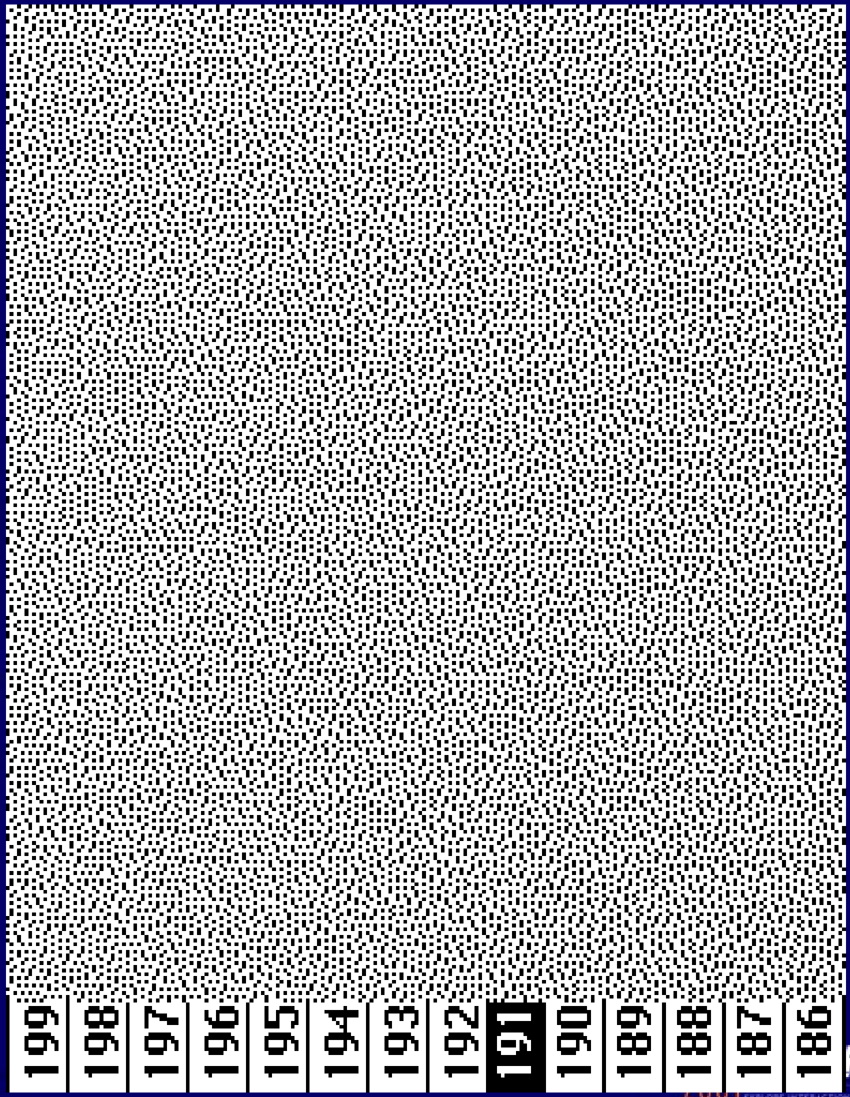
Results

Std. Floyd-Steinberg E-D

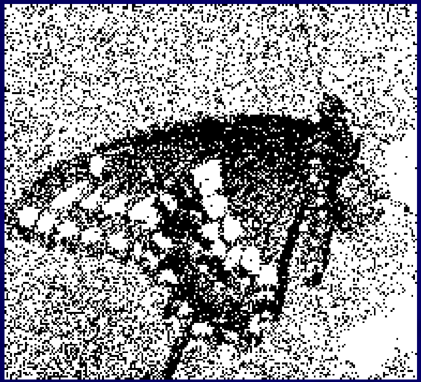


Our Method

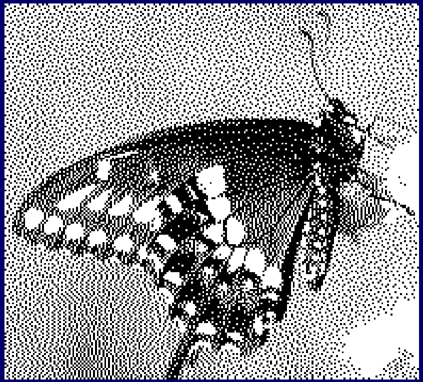


Results

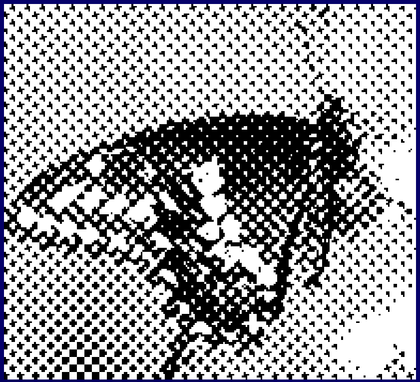
Monte-Carlo



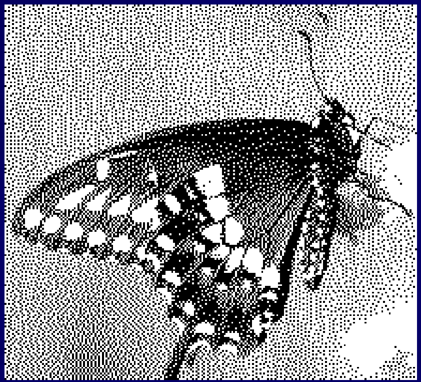
Our Method



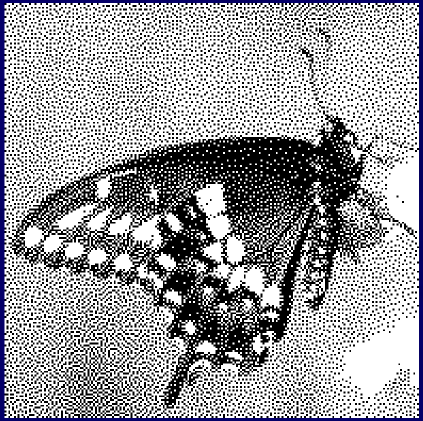
Clustered-Dot Dither



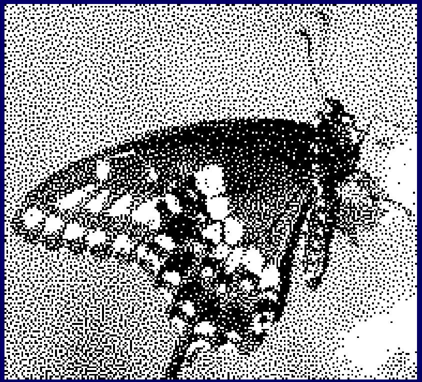
Floyd-Steinberg E-D



DBS-based



Blue Noise Mask



Conclusions

- High-speed Error-Diffusion algorithm
- Good visual quality
- Conceptually simple
- Publicly available on

http://www.iro.umontreal.ca/~ostrom/varcoeffED

- Results of further development will be placed at the same address:

http://www.iro.umontreal.ca/~ostrom/varcoeffED

Future Work

- Taxonomy of Artifacts in E-D
  - Different Artifacts need Different Processing
  - Current Work by P-M Jodoin: Case around ½
- Better Cost Function (Automatic Quality Evaluation)
  - Pathetic Cases (Blue Noise Criterion Does Not Work)
  - Detection of Local Structures (Wavelets, Gabor Functions etc.)
- Systematic Study of Parametric Space Dimensionality
  - Number of Neighbors, Processing Path
- E-D for Network-Oriented Imaging
  - Coherence Through Multiple Depths
  - Temporal Coherence with E-D

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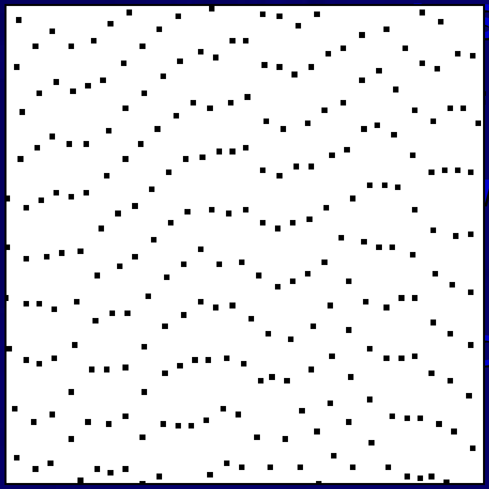
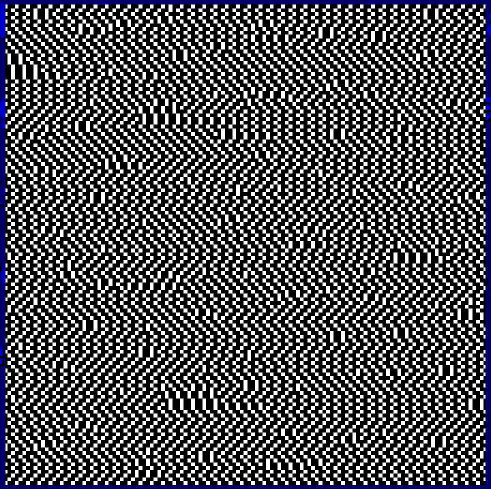
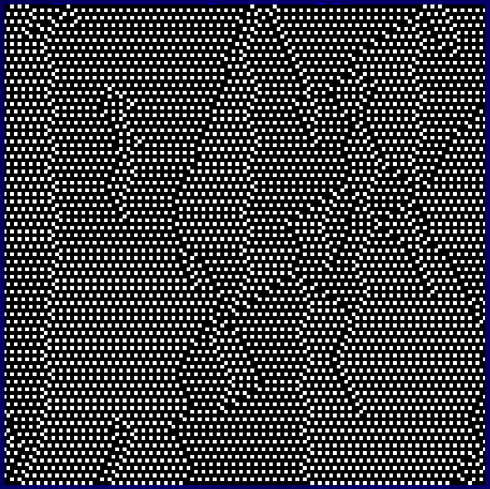
  - **Stud ... ace**
  - **Nei ... ath**
- **ork**
  - **Thr**
  - **Cohe**







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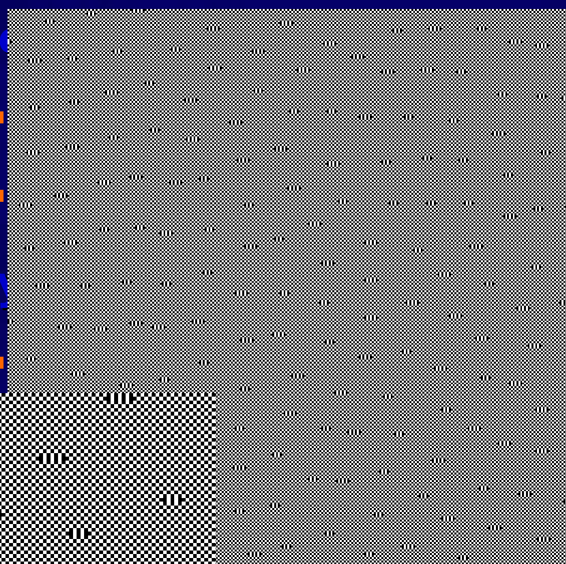
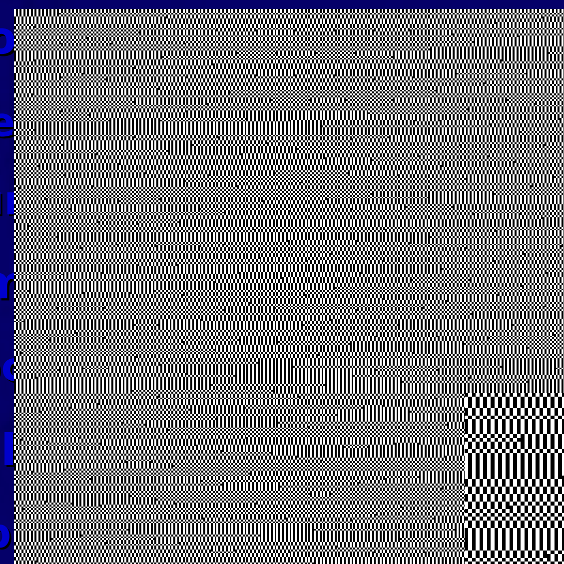
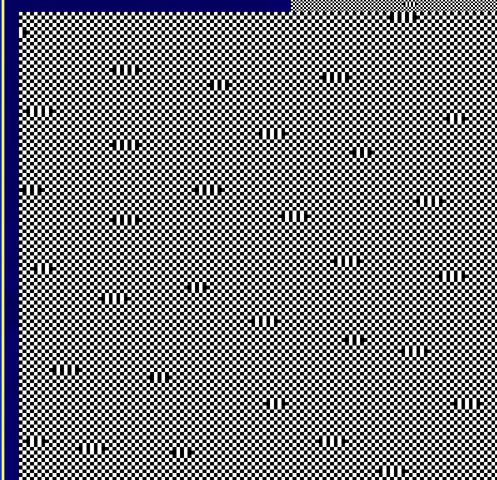
- Be … uto … on)
  - … oise …)
  - … ctu … ctions etc.)
- Sy … ram … nality
  - … Proc
  - … ed I
  - … ltip
  - … al Coherence with E-D

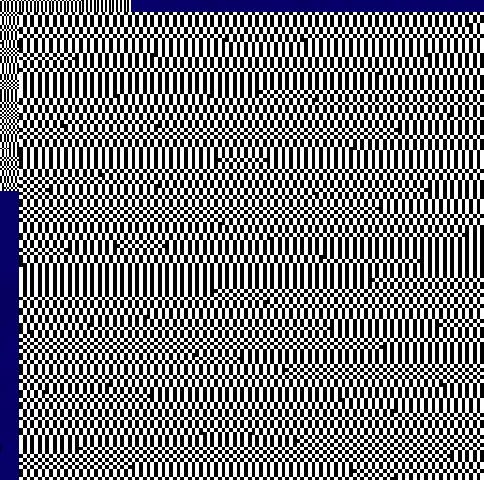






x2 Enlargement

x2 Enlargement



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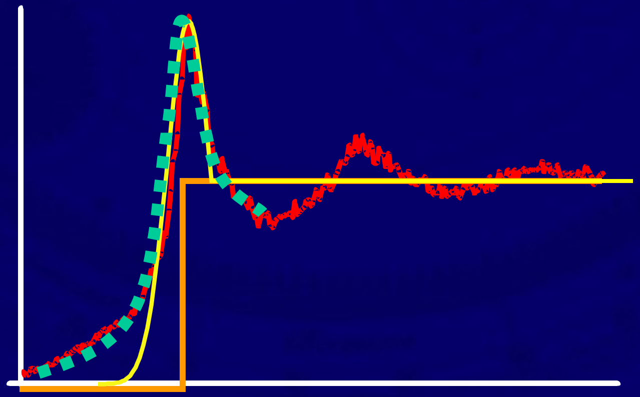

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Future Work

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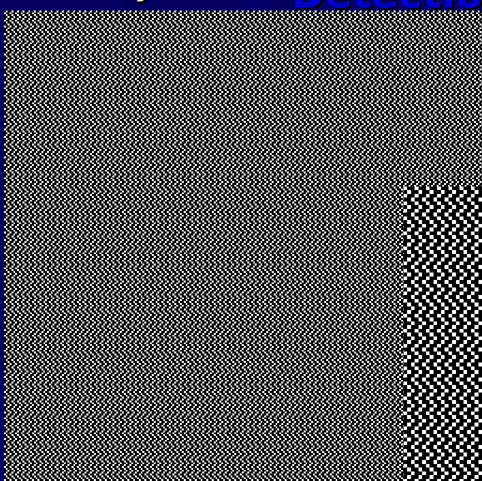
- c Study of Parametric Space Dimensionality
  - of Neighbors, Processing Path
- Imaging
  - le Depths
- E-D

Intensity level = 1/3



x2 Enlargement

Fourier Amplitude Spectrum

Future Work

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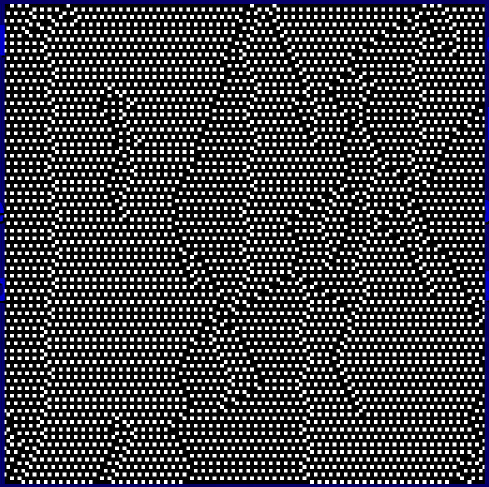
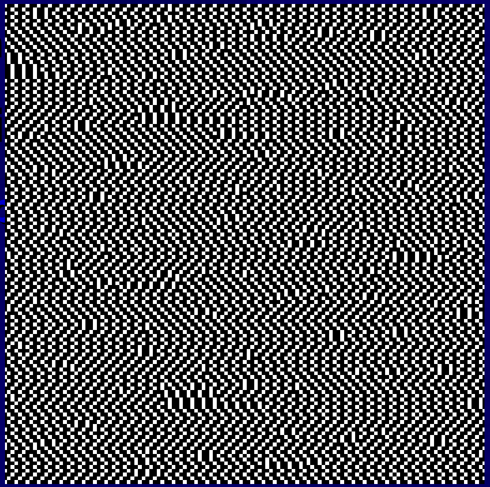
- Systematic Study of Parametric Space Dimensionality
  - Numb … cessin
- E-D for … Imagi
  - Cohe … ple Dep
  - Temp … E-D





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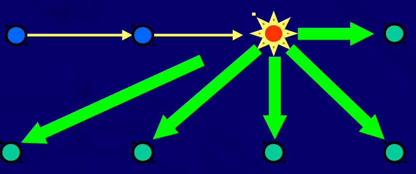

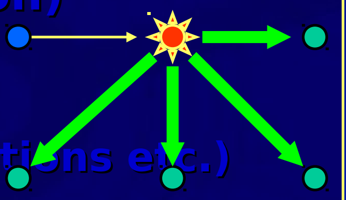

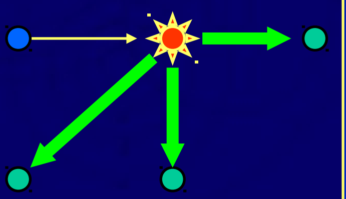

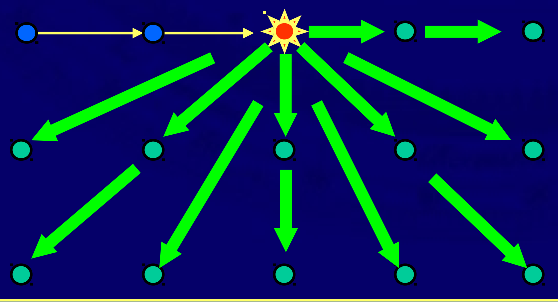

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Thanks

- Bob Ulichney, Jan Allebach, Gabriel Marcu, Reiner Eschbach, Luiz Velho, and Kevin Parker for insightful discussions and for providing sample images of their techniques
- Pierre-Marc Jodoin, Frédo Durand, Justin Legakis, Julie Dorsey, Shlomo Gortler, Roger Hersch, Isaac Amidror, Byong Mok Oh, Max Chen, Neil Stewart, Jean Vaucher, Sebastien Roy, Ovarith Troeung, Aurelien Calais for their help

Thank You